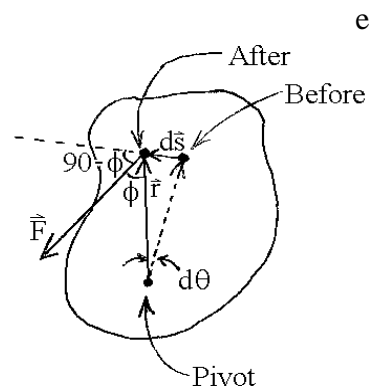


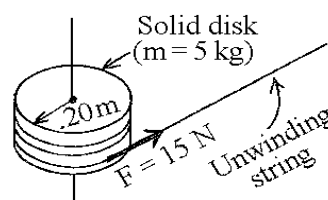
Work done on a rotating object.

(This picture is supposed to stand for any rotating object.)



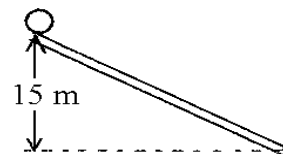
Rotational Kinetic Energy.

Ex. 9-1: The wheel starts from rest. Find  $\omega$  after 10 revolutions. (No friction.)



If an object is moving from place to place and rotating at the same time, like a rolling wheel: Add the translational KE of its center of mass and the rotational energy about the center of mass to get the total.

Ex. 9-2: The solid sphere is released from rest at the top of the ramp. It rolls down without slipping. If no significant energy is lost to friction, what is its speed (in m/s) at the bottom?



Ex. 9-3: A .80 kg disk rolls along a level floor, starting at 4.0 m/s. If the friction force on it is 1.5 N, how far will it go before stopping?

Angular Momentum.

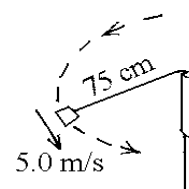
Definition, for a pointlike particle.

Angular momentum of a rigid body.

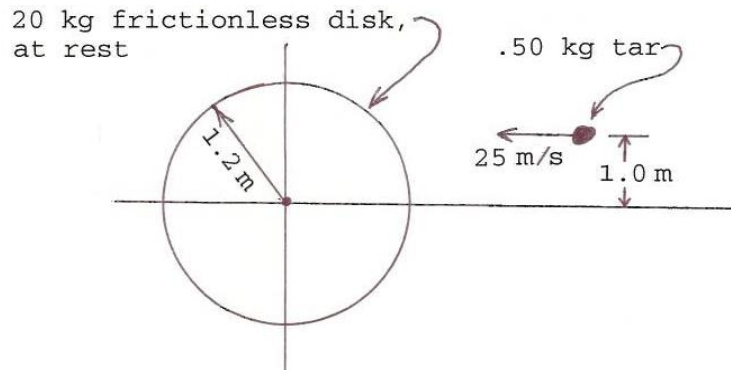
Ex. 9-4: Find the magnitude of Earth's angular momentum, due to its spin.

Conservation of Angular Momentum: If there are no external torques, a system's total  $\vec{L}$  is constant.

Ex. 9-5: A cork initially moves as shown. You then pull the string, shortening the radius to 25 cm. Find the cork's new speed.



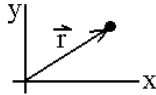

Ex. 9-6: If the tar sticks to the rim of the wheel, what is its final angular velocity?



Summary of basic concepts:

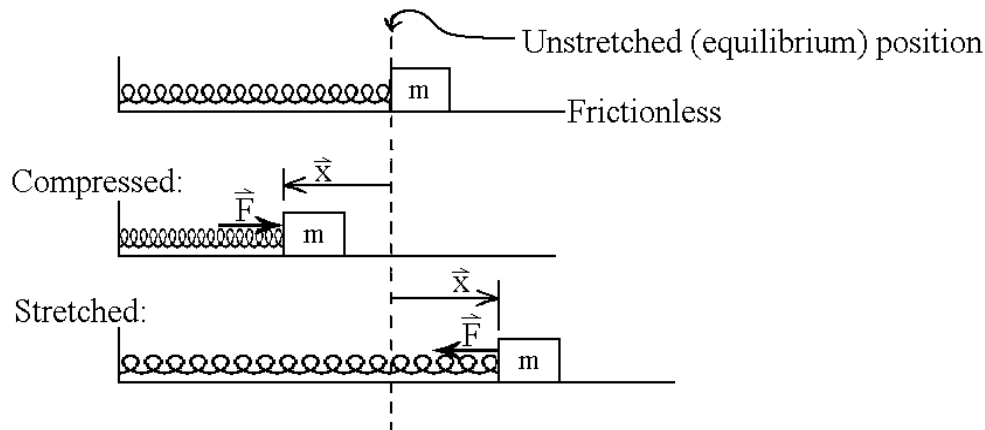
$\frac{d}{dt}$  = rate of change with respect to time

$\frac{d}{dx}$  = rate of change with respect to position

<p>Position: </p> <p>Velocity: <math>\vec{v} = \frac{d\vec{r}}{dt}</math></p> <p>Acceleration: <math>\vec{a} = \frac{d\vec{v}}{dt}</math></p> <p>Momentum: <math>\vec{p} = m \vec{v}</math> ( where m = mass.)</p> <p>Force: <math>\vec{F} = \frac{d\vec{p}}{dt}</math> ( <math>= \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}</math> )</p> <p>Work: <math>dW = \vec{F} \cdot d\vec{r}</math> <math>= d(\frac{1}{2}mv^2)</math></p> <p>Power: <math>P = \frac{dW}{dt}</math></p>	<p>Angle: </p> <p>Angular velocity: <math>\vec{\omega} = \frac{d\vec{\theta}}{dt}</math></p> <p>Angular acceleration: <math>\vec{\alpha} = \frac{d\vec{\omega}}{dt}</math></p> <p>Angular momentum: <math>\vec{L} = \vec{r} \times \vec{p}</math> ( <math>= I \vec{\omega}</math> where <math>I = \int r^2 dm</math> )</p> <p>Torque: <math>\vec{\tau} = \frac{d\vec{L}}{dt}</math> ( <math>= \frac{d}{dt}(I\vec{\omega}) = I \frac{d\vec{\omega}}{dt} = I\vec{\alpha}</math> ) ( <math>= \frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}</math> )</p> <p>Work: <math>dW = \vec{\tau} \cdot d\vec{\theta}</math> <math>= d(\frac{1}{2}I\omega^2)</math></p>
---	--

Notice similarities, for example:  $\vec{p} = m \vec{v}$   
 $\downarrow \quad \downarrow \downarrow$   
 $\vec{L} = I \vec{\omega}$

## Section 10: Springs and Vibration



Hooke's law.

Ex. 10-1: When a 600 N person gets into a certain car, it sinks 1.0 cm. What is  $k$  for its spring system?

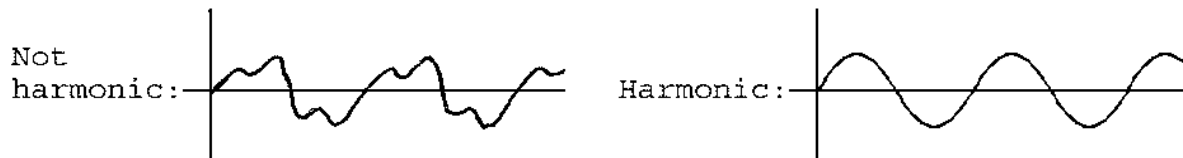
For a mass on a spring, I will show how the expressions for

- position as a function of time,
- velocity as a function of time, and
- acceleration as a function of time

follow from Hooke's law and  $F = ma$ .

This kind of motion is called Simple Harmonic Motion.

Harmonic means motion with a linear restoring force. (It obeys Hooke's law.)

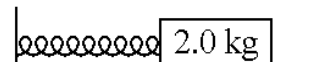


Terminology used to describe vibration: Amplitude, phase angle, period, frequency, angular frequency.

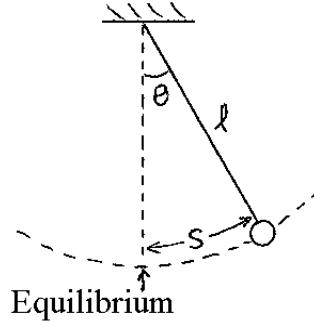
Ex. 10-2: If this system is hung vertically, the spring stretches 39.2 cm.

The 2.0 kg is displaced 20 cm, then released at  $t = 0$ . Find:

- a. the frequency,
- b. the period,
- c. the displacement at  $t = .400$  second.



Pendulum:



The motion is harmonic if the restoring force obeys Hooke's law.

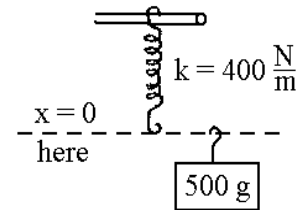
The Restoring force is the tangential component.

I will show the motion to be harmonic for small amplitudes only.

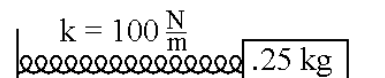
Ex. 10-3: A pendulum 1.00 m long takes 200.6 s to complete 100 cycles. The amplitude is small. Find the local value of  $g$ .

Elastic Potential Energy ( $U_s$ ).

Ex. 10-4: The weight is attached, then released. What is its speed after dropping 2.00 cm?



Ex. 10-5: The oscillator has an amplitude of 30.0 cm. Find its speed when 17.0 cm from equilibrium.



Approximating small changes by differentials:

Ex. 10-6: For 10-5's oscillator, how much does the speed change between  $x = 17.00$  cm and  $x = 17.01$  cm?

## Section 11: Waves

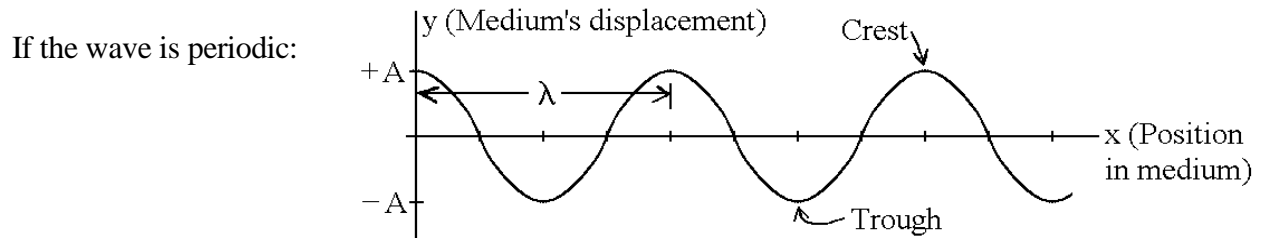
(A wave = a disturbance traveling through some medium.)

Transverse waves: The medium is displaced perpendicular to the direction of propagation.

Examples: light, string waves, water waves (sort of).

Longitudinal waves: The medium is displaced parallel to the direction of propagation.

Examples: compression waves like sound.



Frequency, period, amplitude & displacement mean what they did in sec. 10.

$f$  corresponds to the pitch of sound and the color of light.

$A$  corresponds to the loudness and brightness.

Wavelength ( $\lambda$ ) (Greek "lambda") = distance from one crest to the next. (in meters)

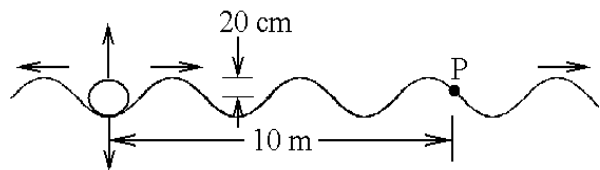
Speed ( $v$ ) = how fast a given crest moves through the medium. (m/s)

Relationship between  $\lambda$ ,  $f$  and  $v$ .

Ex. 11-1: You measure 5.0 ms as a sound's period. Find its (a) frequency, (b) wavelength.

Equation of a one dimensional, sinusoidal (sine-like) wave, moving in the  $+x$  direction.

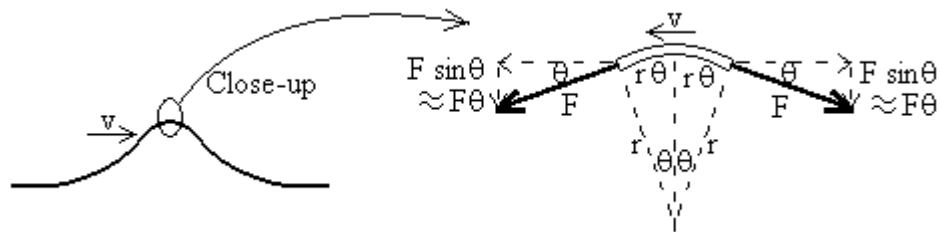
Ex. 11-2: At  $t = 0$ , a lumberjack jumps on a log and starts hopping at 1.25 Hz. The waves travel at 4.00 m/s. Find the displacement of point P at  $t = 5.00$  s.



v (wave's speed): depends on the medium, not the wave. (Different frequencies and amplitudes have about the same speed.)

- Speed of water waves: Depth of the water.

- String waves: On the left is a pulse moving at a speed  $v$  along a string. At right is the small piece of string at the top of the pulse as seen by a moving observer staying even with the pulse.



$F$  = the tension in the string. Such a small piece is indistinguishable from a circular arc, so  $s = r\theta$  gives its length. Also, the piece is in uniform circular motion at this instant with the centripetal force furnished by the downward components of  $F$ . Based on this, I will derive the speed of a transverse wave on a string.

- Speed of sound.

- Speed of light in a vacuum.  $c = 3.00 \times 10^8$  m/s, for all electromagnetic waves:

Radio waves (lowest frequency)

Microwaves

Infrared

Visible light (Roy G. Biv)

Ultraviolet

X-rays

Gamma-rays (highest frequency)

Ex. 11-3: Waves on a piano string need to travel at 240 m/s. If its mass/length is 3.5 g/m, how much tension should it be under?

Intensity.

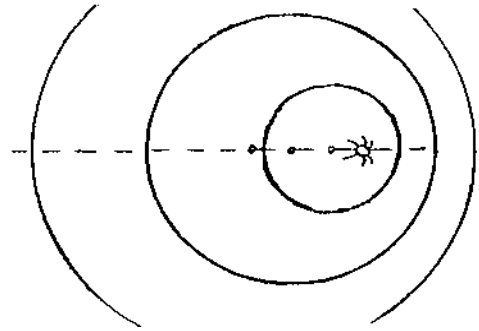
How intensity varies with distance (for a 3 dimensional wave, moving in the open.)

The decibel scale.

Ex. 11-4: A certain distance from a siren, the sound level is 70 dB. What is the sound level if a second identical siren at the same distance is turned on?

Ex. 11-5: 25 m from a machine in an open field, the sound level is 80 dB. What is the sound level 40 m from it?

The Doppler Effect = a shift in  $\lambda$  and  $f$  caused by motion of the source.



Notation:

$v$  = speed of waves,  $v_s$  = source's speed,  $v_o$  = observer's speed

$T$ ,  $f$  &  $\lambda$ : when source is at rest.

$T'$ ,  $f'$  &  $\lambda'$ ; when source is moving.

In a time of one period, the bug moves a distance  $= v_s T$ .

So, in front of the bug  $\lambda' = \lambda - v_s T$ . Behind the bug,  $\lambda' = \lambda + v_s T$ .

From this, and also considering the motion of the observer, the equation for the Doppler effect can be derived.

Ex. 11-6: As a car passes you, the pitch of its horn drops from 500 Hz to 400 Hz. What is the car's speed?

## Section 12:

### Gravitation:

#### The inverse square law.

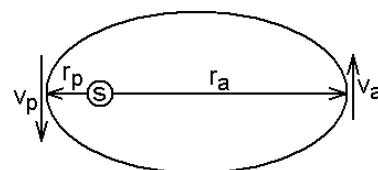
Ex. 12-1: From the following information, find the acceleration due to gravity at the moon's surface. ( $m_m$  = mass of moon,  $m_e$  = mass of earth,  $r$  = radius.)  $m_m = .0123m_e$ ,  $r_m = .273r_e$

(Notice a similar calculation shows a planet's  $m = gr^2/G$ . This is how we know the earth's mass.)

#### Kepler's laws.

As an example of how Kepler's laws were explained and generalized by Newton, I will derive the third law. (For the special case of a circular orbit, and  $m_1 \gg m_2$ )

Ex. 12-2: Given  $v_p$ ,  $r_a$  &  $r_p$ , find  $v_a$ .



Ex. 12-3: A binary star system has a period of 14 years, and an average separation of  $9.5 \times 10^8$  km. Find the total mass.

The temperature of a substance depends on the average kinetic energy of its molecules.

Celsius scale:  $0^\circ\text{C}$  = freezing point of water,  $100^\circ\text{C}$  = boiling point of water

Absolute zero = the minimum possible temperature =  $-273.15^\circ\text{C}$

Kelvin scale:  $0\text{ K}$  = absolute zero,  $273.15\text{ K} = 0^\circ\text{C}$ ,  $373.15\text{ K} = 100^\circ\text{C}$

#### Pressure.

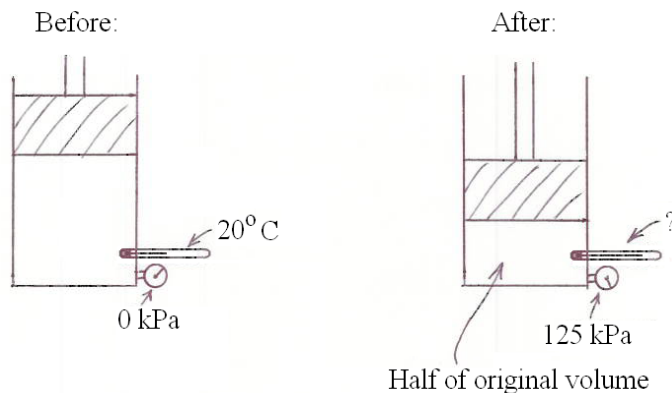
Ex. 12-4: Find the force the atmosphere exerts on a  $7000\text{ cm}^2$  window. (70 cm by 100 cm)

Absolute vs. gauge pressure.

The ideal gas law.

The set of values for  $P$ ,  $V$ ,  $n$  &  $T$  is called the state of the gas.

Ex. 12-5: Find the final temperature reading:



Review of Sec. 9 – 12

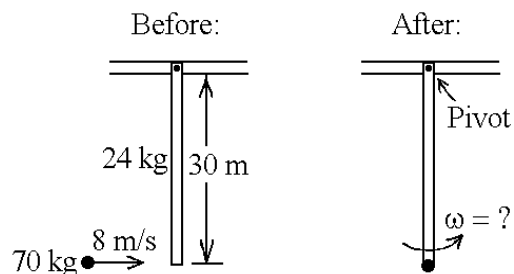
As usual, the best 4 out of 5 count, for 25 points each.

1. To what altitude (above earth's surface) would you have to travel to weigh exactly half of what you do at the surface?

Ans:  $2.64 \times 10^6$  m

2. Tarzan, in the process of swinging from vine to vine, is moving horizontally through the air at 8.0 m/s. His mass is 70 kg. He then grabs a 24 kg vine 30 m long, as shown. Assuming the vine acts like a thin rod pivoted at its upper end, what is its angular velocity just after he grabs it?

Ans: .239 rad/s



3. A particle executes simple harmonic motion with an amplitude of 3.00 cm. At what displacement from the midpoint of its motion will its speed equal one half of its maximum speed?

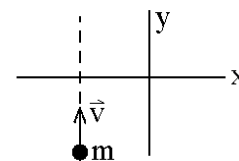
Ans: 2.60 cm

4. You're walking away from a buzzer in a large open field, which is otherwise completely quiet. When 3.0 m from the buzzer, the sound level is 36 dB. What is the maximum distance from the buzzer where you can still hear it? Assume the threshold of hearing is 0 dB =  $1.0 \times 10^{-12}$  W/m<sup>2</sup>.

Ans: 189 m

5. Short answer, 5 points each:

a. What is the direction of this particle's angular momentum vector,  $\vec{L}$ ? (+x, -x, +y, -y, +z or -z?)



b. People sometimes confuse sound waves with radio waves. Explain the difference between them.

c. At which point(s) of a harmonic oscillator's motion is its acceleration the greatest?

d. If the sun was four times as massive as it actually is, and the earth still orbited at the same distance, what would its orbital period be? (Earth's mass is negligible compared to the sun's.)

e. What is an ideal gas? It's a gas whose molecules don't \_\_\_\_\_. (Name at least one thing which its molecules are assumed not to be or do.)

Review of Sec. 9 – 12

As usual, the best 4 out of 5 count, for 25 points each.

1. A spring of stiffness  $k = 15.0 \text{ N/m}$  is hanging vertically. A  $.250 \text{ kg}$  mass is attached to its lower end and released from rest. How far will the spring stretch before the mass stops falling and begins to rise?

Ans: 32.7 cm

2. The two stars in a binary system have a total mass of  $2.30 \times 10^{30} \text{ kg}$  and orbit each other with a period of 91 days. What is their average separation?

Ans:  $6.22 \times 10^{10} \text{ m}$

3. A basketball (thin spherical shell) has a mass of 600 grams and a radius of 12.0 cm. It is released from rest at the top of a ramp and then loses  $.250 \text{ J}$  to friction while rolling down. If it starts out 20.0 cm higher than the bottom of the ramp, what is its final angular velocity?

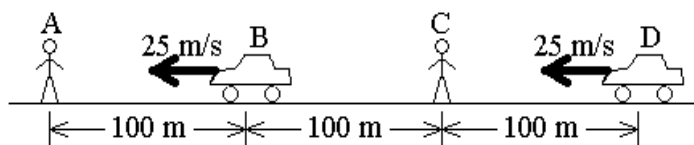
Ans: 11.3 rad/s

4. A member of a marching band has a flute which is perfectly tuned inside a  $20^\circ\text{C}$  building: An A is 440 Hz. She then plays in a parade on a very cold day. The notes are now flat: A is 429 Hz. Each note's wavelength is determined by the length of the flute, which has not changed. What is the temperature of the air in the flute?

Ans: 278 kelvins (or  $5^\circ\text{C}$ )

5. Short answer, 5 points each.

a. The two cars, B and D, are blowing identical horns.



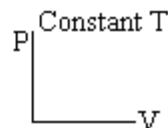
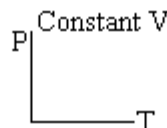
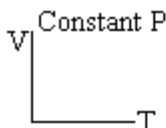
- i. Observer A hears a frequency from B which is \_\_\_\_ (higher than? lower than? the same as?) the frequency from D.
- ii. Observer C hears a frequency from B which is \_\_\_\_ the frequency from D.

b. A simple pendulum is pulled aside so its string makes an angle  $\theta$  with the vertical.

- i. Draw a picture of the pendulum showing the forces on the bob. (The mass on the string.)
- ii. Identify the restoring force which pulls the pendulum back toward equilibrium.

c. The Sun's mass (about 1000 times more) and angular velocity (about one rotation per month compared to Jupiter's one orbit per 12 years) are both much larger than Jupiter's. But Jupiter has about 60% of the solar system's angular momentum while the Sun has under 4%. Explain why.

d. Sketch a line or curve for an ideal gas changing its state at constant pressure, at constant volume and at constant temperature.



e. Compare the speed of sound in air to the speed of sound in a vacuum.

### Section 13:

Thermal expansion.

Ex. 13-1: A 16 gallon tank is full of gasoline at 5°C. If it warms to 30°C, how much runs out?

Heat (Q) = energy transferred from one body to another due to a temperature difference.

Heat units:     1 calorie = amount of heat to raise 1 gram of water 1°C.  
                     1 nutritionist's Calorie = 1 kcal = 1000 cal  
                     1 BTU (British Thermal Unit) = heat to raise 1 lb of water 1°F  
                     Mechanical equivalent of heat: 1 cal = 4.186 J

Specific heat.

Ex. 13-2: 50 g of 90°C water is poured into a 100 g aluminum cup at 20°C. If no heat is lost to the surroundings, find their common final temperature.

Phase changes: Heat energy goes into breaking bonds between molecules, rather than increasing their speed. The temperature stays constant while a substance melts or boils.

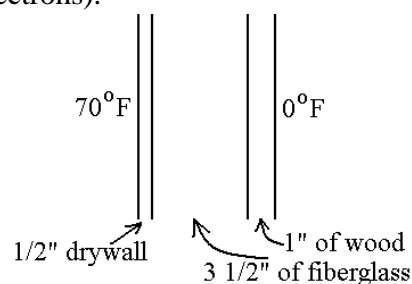
- Heat of Fusion & Heat of Vaporization.

Ex. 13-3: 75 grams of ice at -50°C is in an insulated container. 100°C steam then blows into this container. How many grams of steam are needed to melt all the ice? (Assume it all ends up at 0°C.)

Types of heat transfer (Conduction, Convection & Radiation):

Conduction: Heat transfer by collision of particles (molecules & electrons).

Ex. 13-4: A 20' by 40' house, 8' tall, has walls and ceiling as shown. Assuming no heat flows through the bottom, how many BTU/hour will it take to heat it?



Convection: Heat flow due to flow of heated material. The rising of hot air, for example. It helps make heat go up a chimney, it's involved in the weather, etc.

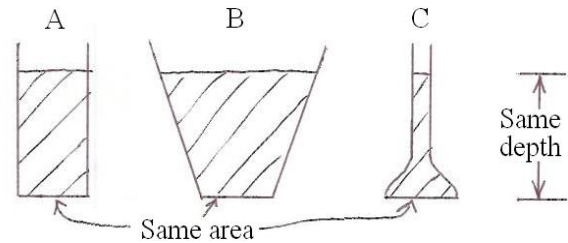
Radiation: Heat flow due to electromagnetic radiation. For example, the sun heating the earth.

Stefan's law. (Also applies to absorption of radiation, where T = temperature of surroundings.)

Ex. 13-5: Find the diameter of a 3700 K star which gives off  $3.6 \times 10^{28}$  watts.

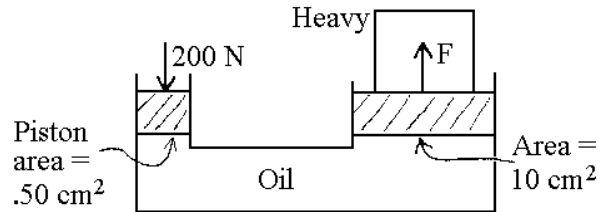


Ex. 14-3: Compare the pressures at the bottoms of the three glasses of water.



Pascal's principle: Additional pressure applied to an enclosed fluid is transmitted to every point in the fluid.

Ex. 14-4: Find the force,  $F$ , put out by the hydraulic press shown.



Buoyancy. Archimedes' principle: Buoyant force = weight of the displaced fluid.

Ex. 14-5: A 100 cm<sup>3</sup> rock, weighing 3.5 N on dry land, is weighed underwater. What does the scale read?

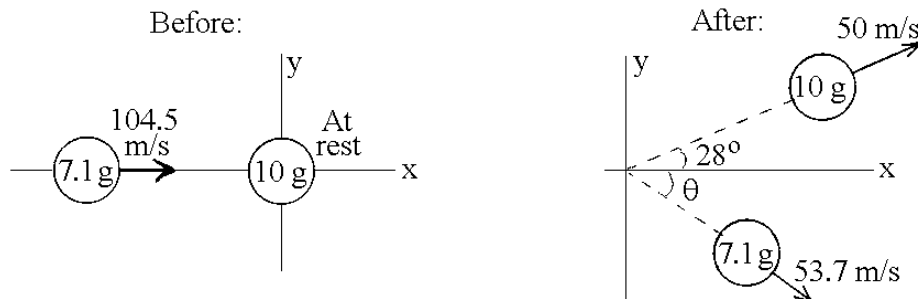
# Review for Final Exam

The test has eight parts each worth 25 points. The best seven you do will be counted. (Perfect score = 175.)

The questions below were picked for being things people often need to work on more, not for similarity to the actual test. You should review the entire course, not just the topics on this sheet.

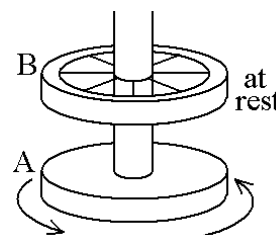
1. A partially inelastic collision takes place as shown. What is the unknown angle,  $\theta$ ?

Ans:  $38.0^\circ$

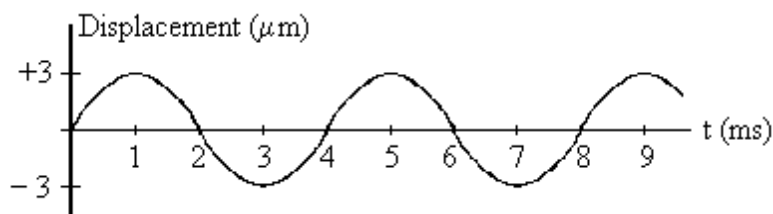


2. A is a solid disk, rotating at  $4.00 \text{ rad/s}$  at the bottom of a lightweight frictionless shaft. B can be thought of as a thin hoop, and can slide without friction along the same shaft. Both have the same mass and radius. If B is dropped onto A from a state of rest,

- What is their final angular speed? (Ans:  $1.33 \text{ rad/s}$ )
- What is the system's final kinetic energy as a percentage of its initial kinetic energy? (Ans:  $33.3\%$ )

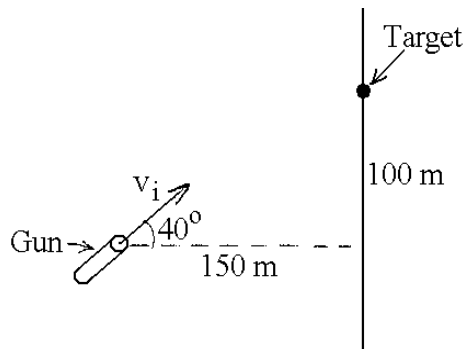


3. The graph shows displacement (in micrometers) as a function of time (in milliseconds) for a sound wave as it passes a microphone. Write the wave function for this sound wave. That is, write  $y$  as a function of  $x$  and  $t$ . Assume that the wave is going down a long pipe toward the right, and that the phase angle is zero.



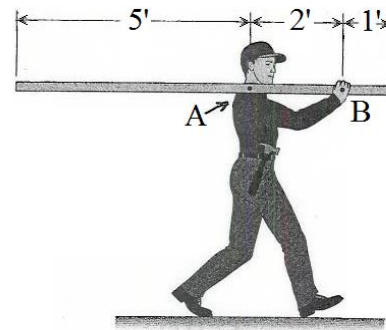
4. Find the speed with which the projectile must be fired if it is to hit the target.

Ans:  $85.2 \text{ m/s}$



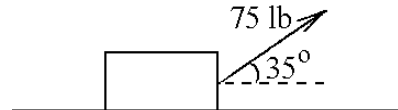
5. A carpenter carries a 12 lb board as shown. What downward force does he feel on his shoulder at A?

Ans: 18.0 lb



6. A box is to be dragged across the floor by a 75 lb force,  $35^\circ$  above the horizontal. If  $\mu_s = .45$ , what is the most the box can weigh and still begin to move?

Ans: 180 lb



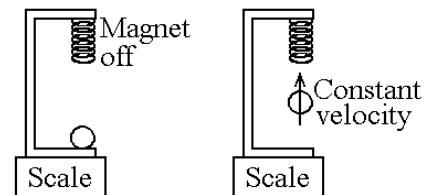
7. Water enters a building through a three inch diameter pipe at  $40.0 \text{ lb/in}^2$ , moving at  $.150 \text{ m/s}$ . All of this water flows on through a half inch diameter pipe in the basement, where the pressure is also  $40.0 \text{ lb/in}^2$ . How much lower in elevation is the half inch pipe than the three inch pipe?

Ans: 1.49 m

8. Short answer, 5 points each:

a. Heat flows from deep within the sun to its surface primarily by the rising of hotter, less dense gas while cooler, denser gas sinks to replace it. Which of the three kinds of heat flow is this an example of?

b. An electromagnet and a steel ball are placed on a non-magnetic scale as shown. The magnet and supports weigh 5 N and the ball weighs 2 N, so the scale reads 7 N in the picture on the left. The magnet is then gradually turned up.



i. When the magnet pulls up on the ball with a 1 N force, what does the scale read?

ii. When the ball is rising with a 2 N force on it, as on the right, what does it read?

c. A baseball is 5.00 m above the ground, traveling straight up at  $10.0 \text{ m/s}$ . Another identical ball is 5.00 m above the ground, traveling straight down at  $10.0 \text{ m/s}$ . Compare the total energy,  $E$ , of one ball to that of the other.

d. i. Sketch a graph of velocity versus time on which there is at least one point where the velocity is zero but the acceleration is not zero.

ii. Sketch a graph of velocity versus time on which there is at least one point where the acceleration is zero but the velocity is not zero.

e. The picture shows the earth orbiting the sun. At the moment shown, what is the direction of its angular momentum vector?

