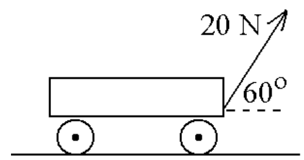
There are two ways to multiply a vector times a vector:

1. The scalar (or "dot") product, and
2. The vector (or "cross") product.

The dot product will be covered now, the cross product in sec. 8.

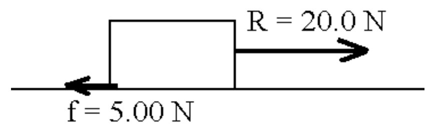
Definition of work. (Notice that only the component of the force parallel to the displacement is thought of as doing work.)

Ex. 5-1: The wagon is pulled 10 m. How much work is done by the 20 N force?



Ex. 5-2: You carry an 80 N bucket of water 100 m over level ground. How much work did you do on it?

Ex. 5-3: The block moves 10 m. Find: (a) the work done by the rope, (b) the work done by friction, and (c) the total work done on the block.



Kinetic Energy

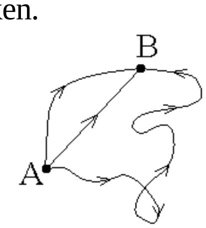
The Work - Energy Theorem

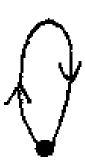
Ex. 5-4: a. Find the kinetic energy of a 70 kg person in a car going 20 m/s.
b. An airbag stops the person in a distance of .50 m. Find the average force.

Ex. 5-5: A 3.0 kg object is moving in a horizontal plane. At t = 2.0 s, $\vec{v} = 12.0\hat{\imath} + 6.0\hat{\jmath}$ m/s. At t = 5.0 s, $\vec{v} = 2.0\hat{\imath} + 4.0\hat{\jmath}$ m/s. Find the work done on it.

A conservative force means one where the work done is independent of the path taken.

For example, if an object moved from point A to point B while a conservative force acted, the total work would add up to the same thing for each path.

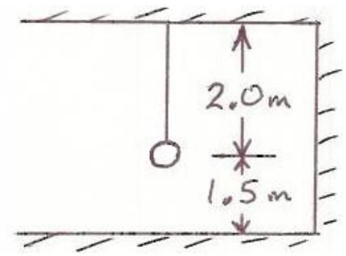




Consider the case where A and B are the same point. One way to get an object from A to B is to not touch it, which does no work. So the work for any other path, like that shown, is also zero: All the energy you put in going in one direction comes back out going the other way. Energy is conserved. (A force like this can be used to store energy.)

Work done by any conservative force can be represented by a type of potential energy;
- Elastic potential energy. (See section 10.)
- Electrostatic potential energy. (See phy 132.)
- Gravitational potential energy. (Covered now.)

Ex. 5-6: Find U_g for the 3.0 kg object.



(Notice: The h in mgh can be measured from any level.)

Summary:

Energy = the capacity to do work.

Kinetic Energy = the work an object could do due to its motion.

Potential Energy = the work an object could do due to its position.

(KE and U are the mechanical forms which energy can take. Non-mechanical forms include chemical energy, heat, etc.)

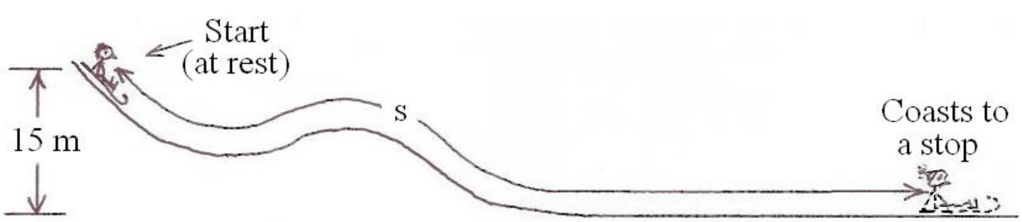
More on the work - energy theorem.

Non-conservative forces just transform mechanical energy into non-mechanical forms. or vice versa. For example, friction transforms kinetic energy into heat. So,

Total energy (mechanical + thermal + chemical + …) of an isolated system is constant.
(Conservation of Energy)

Ex. 5-7: A book falls 2.5 m from rest. Find its final speed. (No air friction.)

Ex. 5-8:



The total mass of the child and sled is 55 kg. If the average force of friction is 83 N, what is the distance, s?

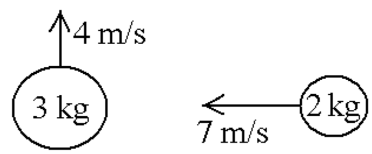
Section 6: Power/ Newton's Third Law & Momentum

Formulas and units for power.

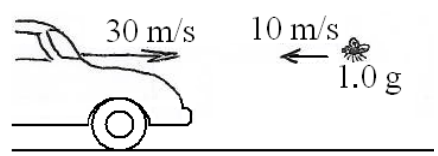
Ex. 6-1: How much power does a crane have to put out to lift 1000 kg 20 m in 10 seconds, at constant speed?

Definition of Impulse,
Definition of Momentum,
The Impulse-Momentum Theorem.

Ex. 6-2: Find this system's momentum:



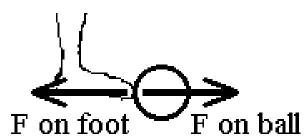
Ex. 6-3: The collision lasts .002 s. The bug ends up stuck to the car. Find the impulse and the average force on the bug.



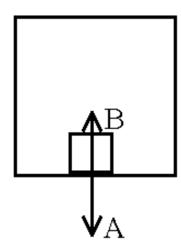
Newton's third law: **If A exerts a force on B, then B exerts an equal and opposite force on A.**

(Called an action - reaction pair.)

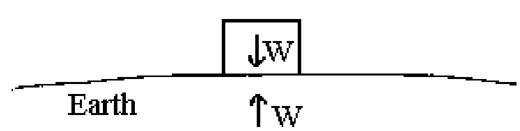
Example: Kicking a ball.



Ex. 6-4: A is the force of the box pushing down on the floor. B is the force of the floor pushing up on the box. When the elevator is in equilibrium, A equals B. How do A and B compare when the elevator is accelerating upward? (A = B, A > B, or A < B ?)



Example: The reaction to Earth's gravitational force on you is your gravitational force on the Earth. (Not the normal force.)



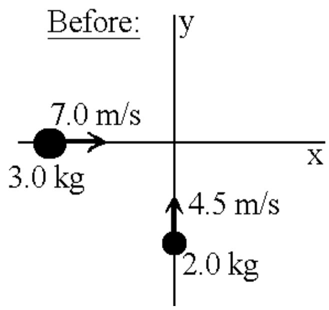
Notice:
- The reaction does not act on the same body as the action.
- Both are the same kind of force. (For example, the reaction to a gravitational force is another gravitational force.)

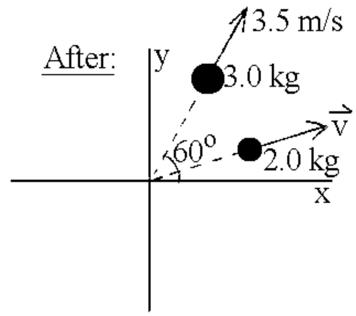
Conservation of momentum follows from Newton's third (and second) law.

Ex. 6-5: A 4.00 kg gun, initially at rest, fires an 8.00 g bullet at 500 m/s. Find the gun's velocity of recoil.

(The same effect is responsible for rocket propulsion.)

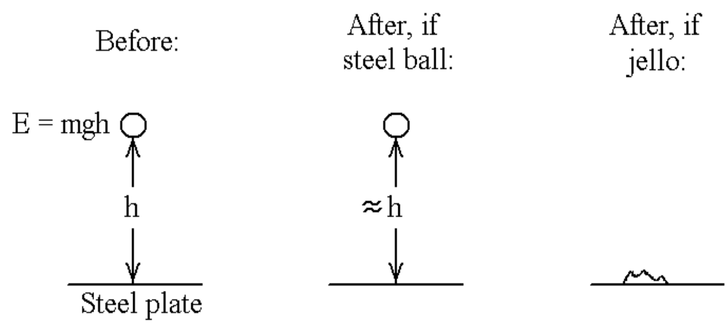
Ex. 6-6: Find the final velocity of the 2 kg:





Mechanical energy, E, need not be conserved in collisions.

For example, drop something on a hard steel plate. E is conserved for a hard steel ball, but not for a piece of jello. As the jello is deformed, internal friction converts E into heat.



Elastic collision: One where E is conserved.
Inelastic collision: One where E is not conserved.
(The most extreme case is a perfectly inelastic collision: one where they stick together.)

mv is conserved in both cases.
There is no conservation of 1/2 mv^2 unless it's elastic.

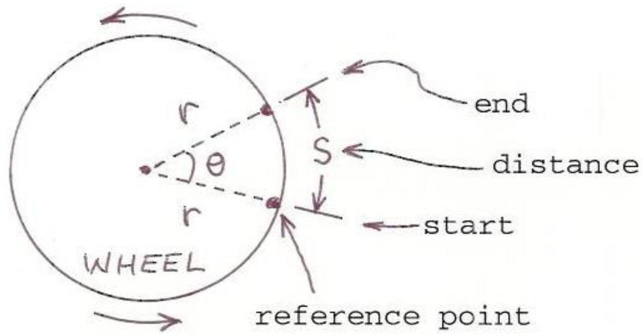
Ex. 6-7: If the collision is elastic, what are v_1 and v_2?

Section 7: Rotation:

How far an object has turned is described by an angle, θ.

Units for θ: 1 revolution = 360° = 2π radians

θ in rad. is defined as $\theta = \frac{s}{r}$ (or $\boxed{s = r\theta}$, θ in radians.)



Notice, being a ratio, a radian is dimensionless:

Ex: $\theta = \frac{s}{r} = \frac{10\ cm}{5\ cm} = 2.0$

(Written 2 rad to distinguish it from 2°.)

Ex. 7-1: A reel in a tape cassette makes 20 revolutions. Its radius is 1.5 cm. How much tape passes through the player?

Angular velocity.

Ex. 7-2: It takes exactly 3 minutes for a record player to make 100 revolutions. Find ω in rev/min.

Ex. 7-3: As a fan coasts to rest, its $\theta = (50t - 7t^2)$ rad. Find ω at t = 3.0 s.

Relationship between v and ω.

Ex. 7-4: A lawn mower blade 30 inches long turns at 2000 rpm. Find the speed of its tip in feet per second.

Angular acceleration.

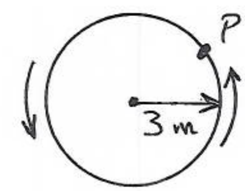
Ex. 7-5: If $\theta = (50t - 7t^2)$ rad, find α at t = 3.0 s.

Relationship between a_t and α.

a_t = tangential acceleration = component of $\vec{a}$ along the direction of motion. Caused by change in $\vec{v}$'s magnitude.

a_c = centripetal acceleration = component of $\vec{a}$ across the direction of motion. Caused by change in $\vec{v}$'s direction. (sec 4)

Ex. 7-6: This is turning at 1.2 rad/s, and accelerating at .85 rad/s^2. Find the radial and tangential components of point P's acceleration.



For uniformly accelerated rotational motion:

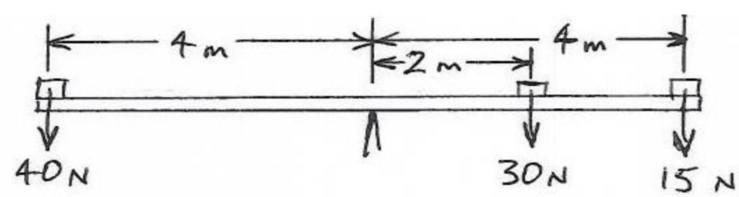
$\omega_f = \omega_i + \alpha t$ $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\Delta\theta = \omega_{av}\, t$ $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$ $\omega_{av} = \frac{1}{2}(\omega_i + \omega_f)$

Ex. 7-7: A tire (r = .35 m) starts to roll downhill. 30 s later, it's going 7.0 m/s. Find its (constant) angular acceleration, and the angle it has turned through.

Torque. Torque is to rotation what force is to linear motion. Torque is found by multiplying the force by the amount of leverage it has. (Counterclockwise is positive, clockwise is negative.)

Ex. 7-8: Find the net torque.

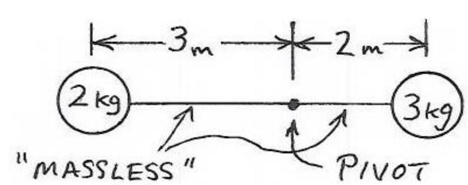


The torque from an object's weight is the same as if the weight was concentrated at a point called the center of gravity. For a uniform object, this is just its geometric center.

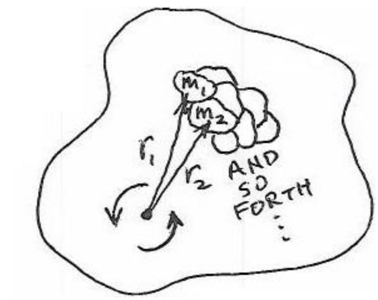
Rotational counterpart of Newton's second law.

Moment of inertia. (The moment of inertia of a pointlike particle is mr^2. For a system of pointlike particles, add the moments of inertia of its parts.)

Ex. 7-9: 60 N·m is applied to this system. Find its angular acceleration.



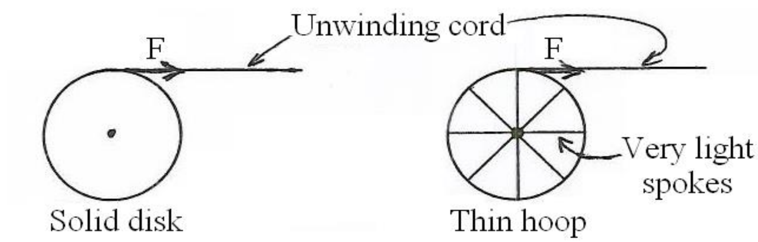
For a solid object, consider it to be many small particles stuck together. Add up their mr^2's to get the approximate I for the object. The approximation becomes exact if you let the number of small particles approach infinity, and the size of each particle approach zero. (Because mr^2 is I for a <u>pointlike</u> particle.)



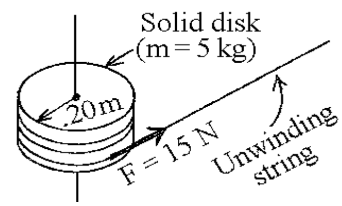
$I = \int r^2 dm$

From this general definition, the specific formulas for different shapes can be derived. (See formula sheet.) Notice that I is larger when mass is located farther from the axis of rotation.

<u>Ex. 7-10</u>: Both wheels have the same mass and radius, and start at rest. The same force acts for the same time. Compare their final angular speeds.



<u>Ex. 7-11</u>: The wheel starts from rest. Find ω after 10 revolutions. (No friction.)

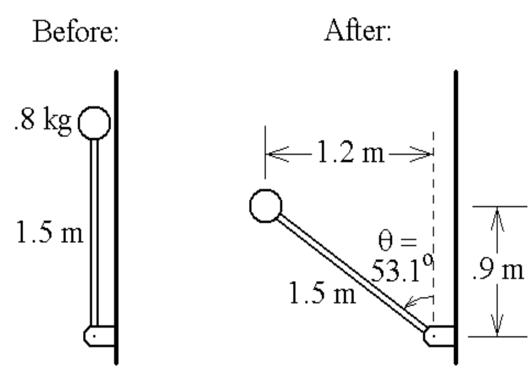


<u>Rotational Kinetic Energy</u>: As explained more fully in section 9, $KE_R = 1/2\ I\omega^2$.

<u>Ex. 7-12</u>: A pointlike 800 gram mass swings on the end of a lightweight rod, as shown. In the "after" picture, what are

(a) the angular acceleration?

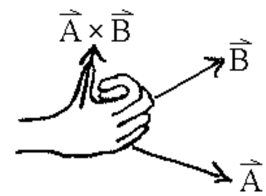
(b) the angular velocity? (Why can't you use $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$?)



Section 8: Torque & Static Equilibrium

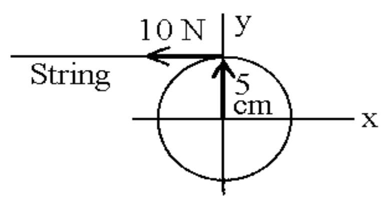
The vector (or "cross") product.

Its direction is given by the right hand rule: Draw the vectors with their tails together, forming an angle. Wrap your fingers around its vertex, pointing from the first vector toward the second. Thumb points in direction of $\vec{A} \times \vec{B}$.



Torque = $\vec{r} \times \vec{F}$. Its direction is along the axis of rotation in a right-handed sense. (Fingers wrap around the axis, pointing in the direction you're trying to make it turn; thumb points in direction of $\vec{\tau}$.)

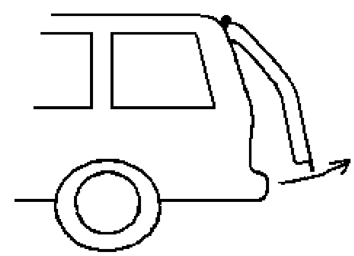
Ex. 8-1: Find the torque on this top.



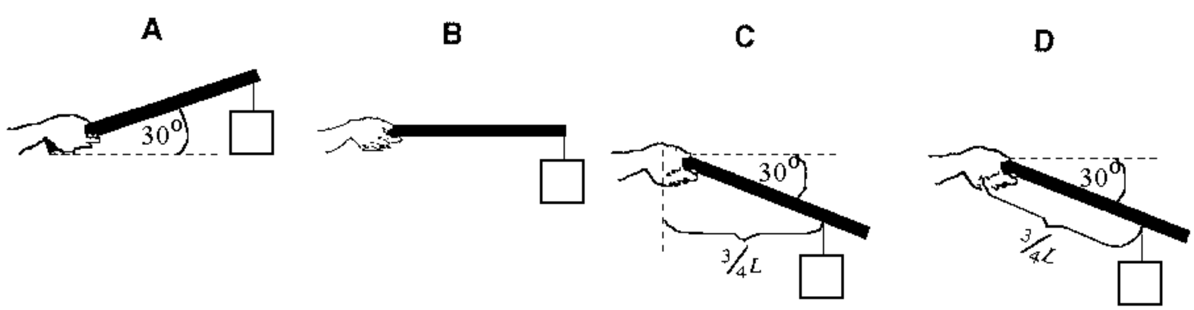
Example of the vector nature of torque: Precession of a gyroscope.

(If the problem is in a plane not 3 dimensions, you usually don't bother writing the $\hat{k}$ on every term. Just use + for counterclockwise, and - for clockwise.)

Ex. 8-2: (a) As the hatch on the back of this vehicle is opening, does the torque from its weight increase, decrease or stay the same?

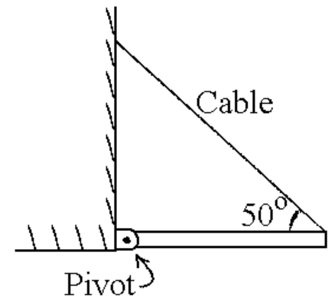


(b) In each picture, a student is holding the end of a stick of length L whose weight is negligible compared to that of the 1 kg mass suspended from it. Rank the difficulty of holding the stick in the position shown from greatest to least.



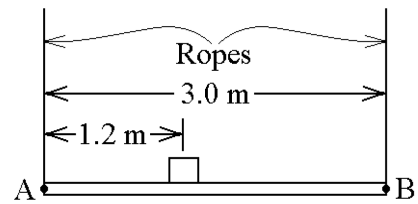
The two conditions for static equilibrium. (Remaining in a state of rest.)

<u>Ex. 8-3</u>: The tailgate weighs 200 N and is 80 cm long. How large is the tension in the cable? What are the x and y components of the force at the hinge?

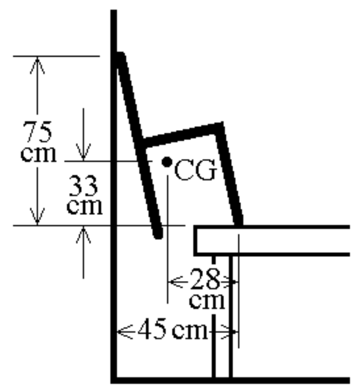


Note: $\Sigma\tau$ will equal 0 about any choice of "pivot."

<u>Ex. 8-4</u>: Find the tension in each rope. The box weighs 150 N, and the board weighs 200 N.



<u>Ex. 8-5</u>: You work on this one, in class. Ask me or another student for help if you need it.



The chair weighs 50 N. The front legs rest on a table, its back leans on a frictionless wall. Find (a) the force from the wall, (b) the x component of the total force on the front legs, and (c) the y component of the total force on the front legs.

Answer: (a) 18.7 N (b) & (c) to be announced.

Review of Sec. 5 – 8:

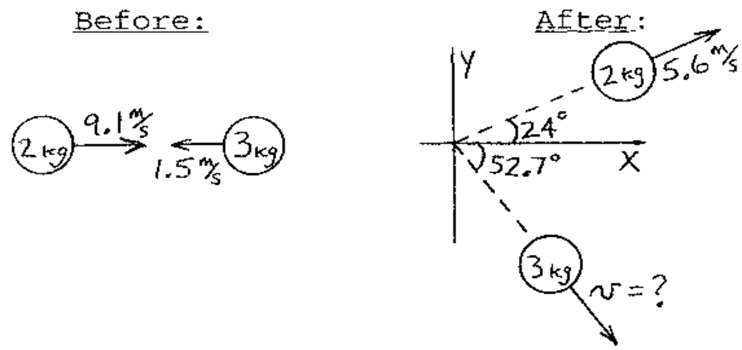
As usual, the best 4 out of 5 will count, for 25 points each. These are not necessarily similar to questions on the actual test; you should review everything.

1. A 7.5 N·m torque acts on a 3.0 kg solid sphere with a 35 cm radius for seven seconds. If it starts from rest, how many radians does it turn?

Ans: 1250

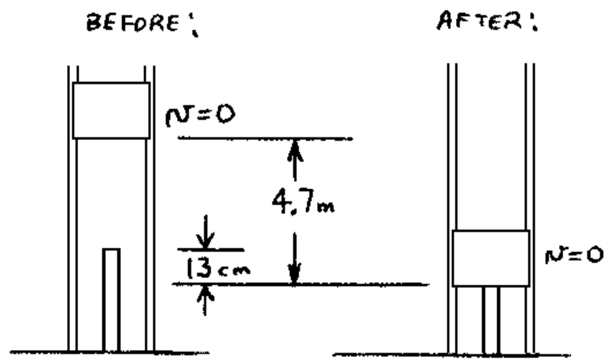
2. Two objects collide as shown. What is the final speed of the 3.0 kg object?

Ans: 1.91 m/s



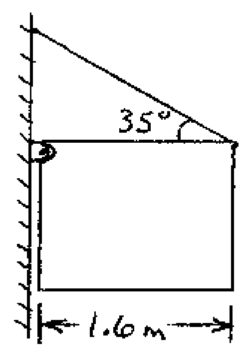
3. The hammer of a pile driver has a mass of 230 kg, and on impact drives the pile 13 cm deeper into the ground. If it is released 4.7 m above its final stopping point, what is the average force it exerts on the pile? (You may use any method you want, but this is easiest using the work - energy theorem.)

Ans: 81.5 kN



4. The uniform sign is supported by a pin at the upper left corner, and a cable at the upper right. If the sign weighs 190 N, how large is the tension in the cable?

Ans: 166 N



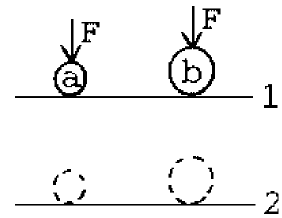
5. Short answer, 5 points each:

a. A large, fast moving truck drives into the back of a small, slow moving car. During the collision, is the force on the truck from the car

i. in a forward direction, or toward the back?

ii. larger, smaller, or the same size as the force on the car from the truck?

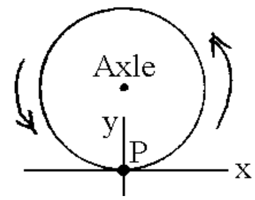
b. Two frictionless pucks start at rest at line 1. Puck b is more massive than a. Identical constant forces are applied until they reach Line 2. When they reach Line 2, which has more kinetic energy, or are they the same?

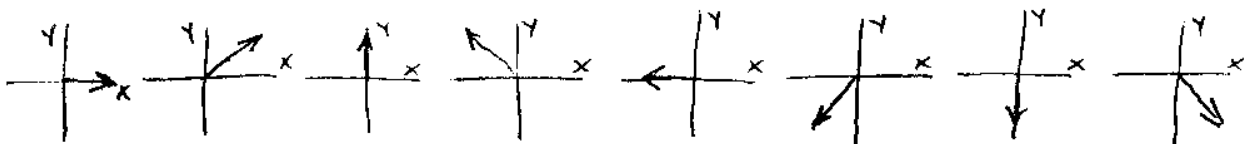


c. Which of the two pucks from the previous question has more momentum at line 2, or are they the same?

d. $\hat{\imath} \times \hat{k}$ = ____.

e. The wheel is spinning counterclockwise with an increasing angular velocity. What is the direction of point P's acceleration vector? (Select one of the following as your answer:





More Review of sec. 5 – 8:

As usual, the best 4 out of 5 will count, for 25 points each. These are not necessarily similar to questions on the actual test; you should review everything.

1. A 2.50 kg grinding wheel is a solid disk with a radius of 15 cm. It is supposed to turn at 480 revolutions per minute.

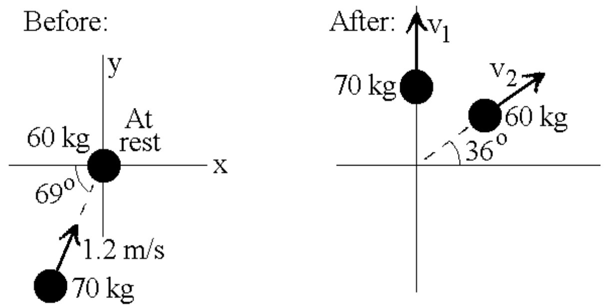
a. Find its moment of inertia.

b. How large is the torque needed to get it up to speed in 5.00 s, starting from rest?

Ans: .0281 kg·m^2, .282 N·m

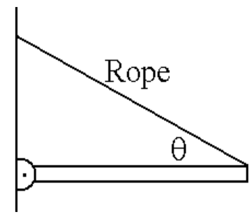
2. A clumsy astronaut, floating in a space station, bumps into another astronaut as shown. Find their final speeds.

Ans: .808 m/s, .620 m/s



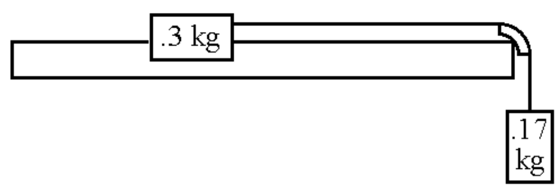
3. The uniform drawbridge is 9.00 m long and weighs 1100 N. It is supported by a single rope, as shown, which will break if the tension in it passes 1300 N. With no other weight on the drawbridge, what is the smallest θ can be?

Ans: 25.0°



4. A .300 kg glider on a level air track is connected to a hanging .170 kg mass by a light cord as shown. If the system starts from rest and there is no friction, what is its speed after traveling 1.40 m?

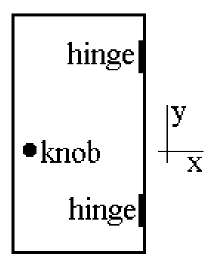
Ans: 3.15 m/s



5. Short answer, 5 points each.

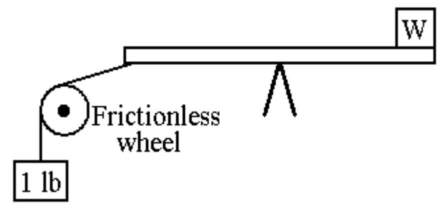
a. A lump of jello has some potential energy as you hold it a distance h above the floor. What kind of energy has this become after you drop it and it is lying at rest on the floor?

b. You are standing in front of this door which lies in the xy plane. If you pull on the doorknob, creating a force in the +z direction, which way does the torque vector point?



c. Identical bullets are fired at the same speed from different guns, initially at rest. As the guns recoil due to being fired, the heavier gun gets ____ (more? less? the same?) momentum compared to the lighter one.

d. The see-saw is balanced. The weight W is ____ (more than? less than? equal to?) 1 pound.



e. Gear A drives gear B.

i. Is the speed of a tooth on the edge of gear A more, less or the same as a tooth on gear B?

ii. Is the angular speed of gear A more, less or the same as gear B's?