Phy 131 - Assignment 9

A. A force directly toward the center produces no torque, so angular momentum is conserved. Explosions involve non-conservative forces, so $\frac{1}{2}I\omega^2$ is not conserved.

a. The final angular momentum equals the initial angular momentum. $L_i = I\omega_i$

I of a sphere: $I_i = \frac{2}{5}MR^2 = (.4)(5 \times 10^{30}\text{ kg})(1 \times 10^{7}\text{ m})^2 = 2.00 \times 10^{44}\text{ kg}\cdot\text{m}^2$

$\omega_i = \left(1\,\frac{\text{rev}}{\text{day}}\right)\left(\frac{1\text{ day}}{86\,400\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 7.272\text{x}10^{-5}\text{ rad/s}$

$L_i = (2.00 \times 10^{44})(7.272\text{x}10^{-5}) = 1.454 \times 10^{40} \approx \boxed{1.45 \times 10^{40}\text{ kg}\cdot\text{m}^2/\text{s}}$

Do (c) next then come back to (b).

c. $L_i = L_f$ $\quad\quad I_f = \frac{2}{5}(5 \times 10^{30}\text{ kg})(1 \times 10^{4}\text{ m})^2 = 2.00 \times 10^{38}\text{ kg}\cdot\text{m}^2$

$1.454 \times 10^{40} = (2.00 \times 10^{38})\,\omega_f$

$\omega_f = \frac{1.454 \times 10^{40}}{2 \times 10^{38}} = \boxed{72.7\text{ rad/s}}$ (One revolution per .0864 second.)

b. $KE_f = \frac{1}{2}I\omega^2_f = \frac{1}{2}(2.00 \times 10^{38}\text{ kg}\cdot\text{m}^2)(72.7\text{ rad/s})^2 = \boxed{5.29 \times 10^{41}\text{ J}}$

B. 1. <u>Sliding</u>. With purely translational motion, all of the potential energy it started with ends up as $\frac{1}{2}mv^2$. When it rolls, part of the initial energy ends up as rotational kinetic energy. If only part ends up as $\frac{1}{2}mv^2$, that means a smaller v than when it all does.

2.

$E_i = E_f$ (No non-conservative forces.)

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$$

$$E = \frac{1}{2}mv^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{5}m\cancel{R^2}\right)\left(\frac{v}{\cancel{R}}\right)^2 + mgh$$

sphere $v = R\omega \Rightarrow \omega = \frac{v}{R}$

$$E = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + mgh$$

$$E = \frac{7}{10}mv^2 + mgh$$

$$\frac{7}{10}\cancel{m}v_i^2 + \cancel{m}gh_i = \frac{7}{10}\cancel{m}v_f^2 + \cancel{m}gh_f$$

$$\frac{7}{10}(4.3\,\tfrac{m}{s})^2 + (9.8\,\tfrac{m}{s^2})(2.5\,m) = \frac{7}{10}v_f^2 + (9.8)(0)$$

$$12.94 + 24.5 = \frac{7}{10}v_f^2$$

$$\frac{10}{7}(37.44) = v_f^2$$

$$v_f = \sqrt{53.49} = \boxed{7.31\,\tfrac{m}{s}}$$

C. While rolling:

total KE = $.5mv^2 + .5I\omega^2 = .5mv^2 + .5(2/3\ mR^2)(v/R)^2$

$I = 2/3\ mR^2$ for a spherical shell $\omega = v/R$ from $v = R\omega$

KE = $.5mv^2 + 1/3\ mv^2$

KE = $5/6\ mv^2$

$E_i + W = E_f$

$[5/6\ mv_A^2 + mg(0)] + 0 = [5/6\ mv_B^2 + mgh_B]$

Divide through by m:

$5/6\ (3.5)^2 = 5/6\ v_B^2 + (9.8)(.7)$

$10.21 = 5/6\ v_B^2 + 6.86$

$3.35 = 5/6\, v_B^2$

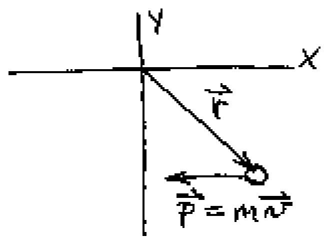
$4.02 = v_B^2$

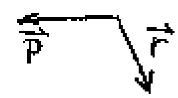
$\boxed{2.00 \text{ m/s}} = v_B$

b. From $v = R\omega$, $\omega = v/R = (2 \text{ m/s}) / (.12 \text{ m}) = \boxed{16.7 \text{ rad/s}}$

c. It will rise until its center of mass stops rising. At that moment, $v = \boxed{0}$

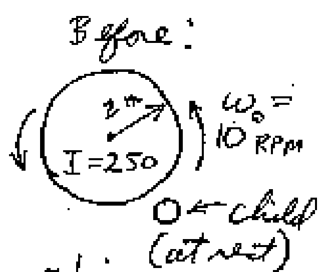
d. Once it's no longer touching the ramp, there is no longer any torque to make it spin faster or slower. It will continue to spin at the same rate the whole time it's in the air. So, the answer is the same as (b). 16.7 rad/s

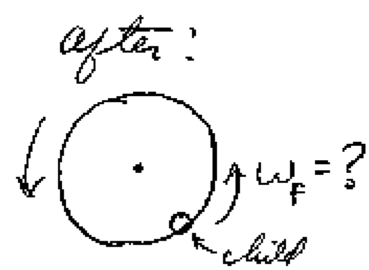




D. 1. Applying the right hand rule to these vectors shows that $\vec{L} = \vec{r} \times \vec{p}$ points in the <u>negative z direction</u>. (Draw the vectors with their tails together. Then, wrap your fingers around the vertex of the angle, pointing from the first vector toward the second one. Your thumb points in the direction of the cross product.)

2.





Cons. of L:

$$I_{MGR}\,\omega_0 + 0 = I_{MGR}\,\omega_f + I_c\,\omega_f$$

merry-go-round

child: $mr^2 = (25\text{ kg})(2\text{ m})^2 = 100 \text{ kg}\cdot\text{m}^2$

$$(250)\left[\left(10\,\frac{\text{REV}}{\text{MIN}}\right)\left(\cancel{2\pi}\,\frac{\text{RAD}}{\text{REV}}\right)\right] = (250 + 100)\left[\left(\omega_f\,\frac{\text{REV}}{\text{MIN}}\right)\left(\cancel{2\pi}\,\frac{\text{RAD}}{\text{REV}}\right)\right]$$

(L = Iω requires the use of radians, but in this case the conversion factor drops out so revolutions works.)

$$2500 = 350\,\omega_f$$

$$\omega_f = \frac{2500}{350} = \boxed{7.14\ \frac{\text{REV}}{\text{MIN}}}$$

E. 1. <u>Sphere first, hollow cylinder last</u>. This is easiest to see for the case where they all have the same mass and radius: The larger the moment of inertia, the more sluggish the object will be. The sphere, 2/5 MR^2, is the least sluggish. The solid cylinder, ½ MR^2 is next. The thin hollow cylinder, $I = MR^2$, reaches the bottom last. (If you work through the details as I did in class, the mass and radius drop out: Any solid sphere wins, and any tube is last.)

2. total KE = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mR^2)(v/R)^2$

$I = \frac{1}{2}mR^2$ for a cylinder, $\omega = v/R$ from $v = R\omega$

KE = $\frac{1}{2}mv^2 + \frac{1}{4}mv^2$

KE = $\frac{3}{4}mv^2$

$E_i + W = E_f$

$\frac{3}{4}mv_A^2 + 190\text{ J} = \frac{3}{4}mv_B^2$ ($U_i = U_f = 0$ because ground is level.)

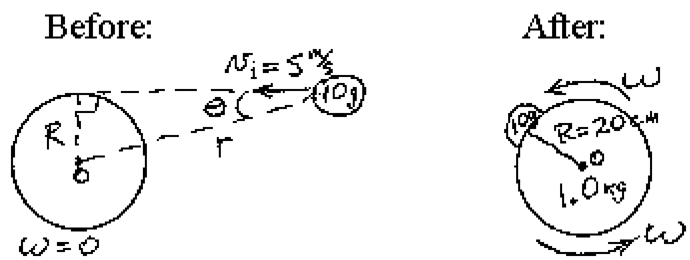
$\frac{3}{4}(50\text{ kg})(1.75\text{ m/s})^2 + 190\text{ J} = \frac{3}{4}(50\text{ kg})v_B^2$

$114.8 + 190 = 37.5\,v_B^2$

$304.8/37.5 = v_B^2$

<u>v_B = 2.85 m/s</u> (ans.)

F.



a) Angular momentum will be conserved about the axis through O, where
$L = L_{sphere} + L_{particle}$.

Before: $L_o = \underset{\text{sphere at rest}}{0} + |\vec{r} \times \vec{p}| = \underbrace{r \sin\theta}_{= R}\, \underbrace{p}_{= mv} = (.2m)[(.010kg)(5\,m/s)]$

$L_o = .01\ \frac{kg \cdot m^2}{s}$

after: $L_f = I_s\omega + I_p\omega = (\frac{2}{5}m_s R^2)\omega + (M_p R^2)\omega$

$= \frac{2}{5}(1kg)(.2m)^2\omega + (.01kg)(.2m)^2\omega$

$= .0160\omega + .0004\omega = .0164\omega = L_f$

$L_o = L_f \Rightarrow .01 = .0164\omega \Rightarrow \omega = \frac{.01}{.0164} = .609756\ldots$

$\approx \boxed{.610\ \frac{RAD}{s}}$

B) $E = E_{sphere} + E_{particle}$

$E_o = \underset{\text{sphere at rest}}{0} + \frac{1}{2}m_p v_p^2 = \frac{1}{2}(.01kg)(5\,m/s)^2 = .125J$

$E_f = \frac{1}{2}I_s\omega^2 + \frac{1}{2}I_p\omega^2 = \frac{1}{2}(.0160)(.61\frac{RAD}{s})^2 + \frac{1}{2}(.0004)(.61\frac{RAD}{s})^2$

from part A

$= .003J$

amount lost $= .125 - .003 = \boxed{.122J}$ ANS