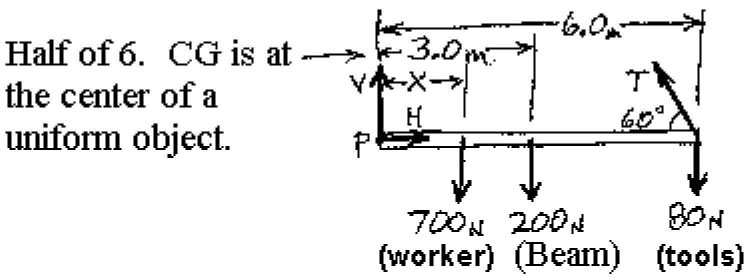
Phy 131 - Assignment 8

A. 1. <u>Yes.</u> Equilibrium means a state of balance, not a state of rest. If the forces and torques on a body balance out to totals of zero, it continues to move as it did originally. (An object at rest remains at rest: Static Equilibrium. An object in motion remains in motion: Dynamic Equilibrium.)

2. The first step is the free body diagram. The equations come from the diagram; if you skip the diagram, you will probably get lousy equations. ("Free body diagram" means a picture of the beam showing all of the forces on it.)



Each arrow on the diagram times r sinθ is a torque. After you do components, each vertical arrow is a term in ΣF_y. Each horizontal arrow is a term in ΣF_x.

$\Sigma \tau_P = 0$

$x = 1\,\text{m}$

$$V(0) + H(0) - (700)(x) - (200)(3\,\text{m}) - (80)(6\,\text{m}) + T\sin 60^\circ(6\,\text{m}) = 0$$

$$-700 - 600 - 480 + 5.196T = 0$$

$$5.196T = 1780 \Rightarrow T = 342.57 = \boxed{343\,\text{N}} \text{ Tension}$$

$\Sigma F_x = 0$

$$H - T\cos 60 = 0 \Rightarrow H = (342.57)\cos 60^\circ = \boxed{171\,\text{N}} \text{ Horizontal Component}$$

$\Sigma F_y = 0$

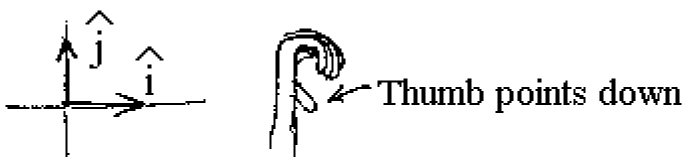
$$V + \underbrace{T\sin 60}_{297\,\text{N}} - 700 - 200 - 80 = 0 \qquad (T = 342.57\,\text{N})$$

$$V - 683 = 0 \Rightarrow \boxed{V = 683\,\text{N}} \text{ Vertical component.}$$

B. 1. The cross product of two vectors is $\vec{A} \times \vec{B} = |A|\ |B| \sin\theta\, \hat{n}$, where $\hat{n}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$.

For the 1st and 4th parts: If a vector is crossed with itself, the angle is 0°; since sin 0° = 0, the cross product is $\vec{0}$ in those cases.

For the others: Since $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are all perpendicular to each other, sin 90° = 1. The magnitudes of all these vectors are also one, (that's what it means to be a unit vector) so the answers are unit vectors. Exactly which unit vector can be determined with the right hand rule. For example, the third one, $\hat{\jmath}$ x $\hat{\imath}$:



ANS: $-\hat{k}$

Wrap your fingers around the origin, pointing from the first vector toward the second. (The cross product is not commutative; pay attention to which vector is on which side of the "x" symbol.) Your thumb points in the direction of$\hat{n}$.

See me if you'd like a demonstration.

So, the answers are:$\vec{0}$, $\hat{k}$, - $\hat{k}$, $\vec{0}$, $\hat{\jmath}$, - $\hat{\imath}$.

2.

See solution to previous problem for free body diagram.

$$\Sigma \tau_p = 0$$

$$V(0) + H(0) - (700)(x) - (200)(3m) - (80)(6m) + T \sin 60^\circ (6m) = 0$$

$$-700X - 600 - 480 + 5.196T = 0$$

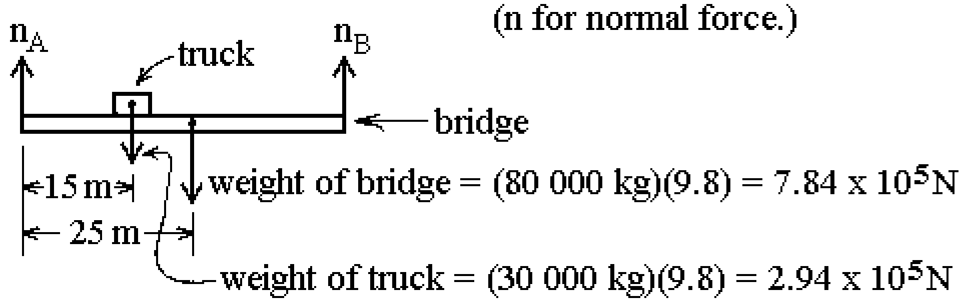
Fill in max. T = 900 N to get max. X

$$-700X - 600 - 480 + 4676 = 0$$

$$-700X + 3596 = 0$$

$$X = \frac{3596}{700} = \boxed{5.14\ m}$$

C.



Only because it's easier to write, I'll have the forces in kilonewtons instead: 784 kN and 294 kN. Taking torques about point A,

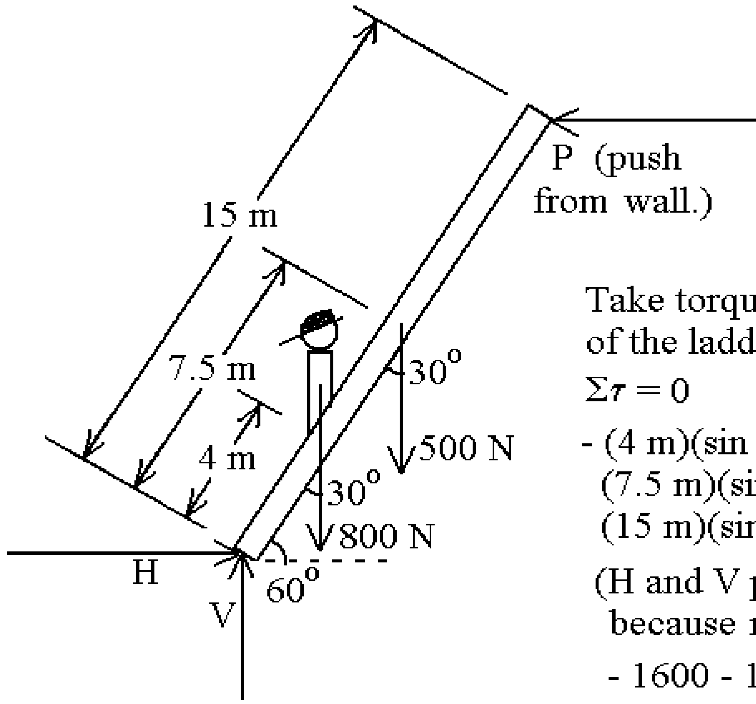
clockwise   counterclockwise

$$\Sigma \tau_A = 0$$
$$(n_A)(0\,\mathrm{m}) - (294)(15\,\mathrm{m}) - (784)(25\,\mathrm{m}) + (n_B)(50\,\mathrm{m}) = 0$$
$$-4410 - 19600 + 50\,n_B = 0$$
$$50\,n_B = 24010\ \mathrm{N{\cdot}m}$$
$$n_B = \frac{24010}{50} = \boxed{480\ \mathrm{kN}}$$

$$\Sigma F_Y = 0$$
$$n_A + 480 - 784 - 294 = 0$$
$$n_A - 598 = 0$$
$$\boxed{n_A = 598\ \mathrm{kN}}$$

(An alternate approach instead of using $\Sigma F_y = 0$ would be to take torques about point B.)

D.



Take torques about the foot of the ladder:

$\Sigma\tau = 0$

$-\,(4\text{ m})(\sin 30)(800\text{ N}) - (7.5\text{ m})(\sin 30)(500\text{ N}) + (15\text{ m})(\sin 60)(P) = 0$

(H and V produce no torque because $r = 0$.)

$-1600 - 1875 + 12.99P = 0$

$12.99P = 3475$

$P = 268\text{ N}$

$\Sigma F_x = 0$

$H - P = 0$

$H = P$

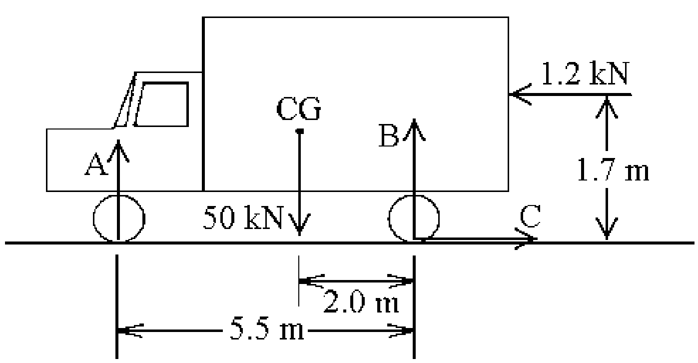
$\boxed{H = 268\text{ N}}$ ans.

$\Sigma F_y = 0$

$V - 500 - 800 = 0$

$\boxed{V = 1300\text{ N}}$ ans.

E. Take torques about the rear wheel (so that the unknown forces there get multiplied by zero):



$\Sigma\tau_{rear} = 0$

$B(0) + C(0) + (1.2)(1.7) + (50)(2) - A(5.5) = 0$

$2.04 + 100 = 5.5A$

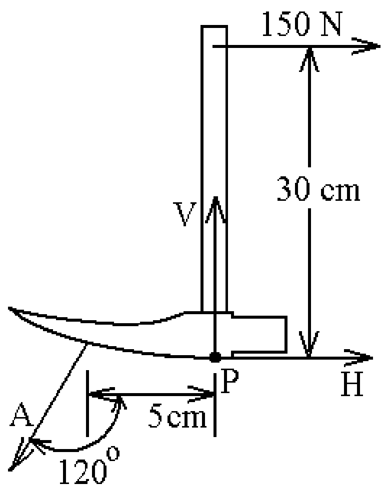
$A = \dfrac{102.04}{5.5} = \boxed{18.6\text{ kN}}$

The easiest way to get B and C is to use forces now. Every vertical arrow on the picture becomes a term in the y equation. Every horizontal arrow becomes a term in the x equation:

$$\Sigma F_y = 0$$
$$A + B - 50 = 0$$
$$18.6 + B - 50 = 0$$
$$B = 50 - 18.6 = \boxed{31.4 \text{ kN}}$$

$$\Sigma F_x = 0$$
$$C - 1.2 = 0$$
$$C = \boxed{1.20 \text{ kN}}$$

F.

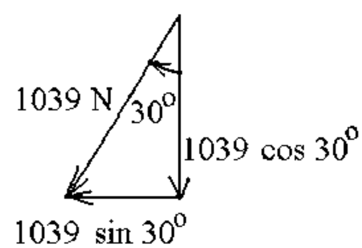


A is the force from the nail.
H and V are the horizontal and vertical components of the force from the point of contact, point P.

$$\Sigma \tau_P = 0$$
$$(5 \text{ cm})(\sin 120^o)(A) - (30 \text{ cm})(\sin 90^o)(150 \text{ N}) = 0$$
$$4.330 \text{ A} = 4500$$
$$A = 1039 \text{ N}$$

Rounding, $\boxed{1.04 \text{ kN}}$ ans.

Components: 1039 N in that direction is equivalent to 900 N down and 520 N left acting together.



$$\Sigma F_x = 0$$
$$-(1039 \text{ N})(\sin 30^o) + H + 150 \text{ N} = 0$$
$$-520 + H + 150 = 0$$
$$H = 370 \text{ N}$$

$$\Sigma F_y = 0$$
$$-(1039)(\cos 30^o) + V = 0$$
$$V = 900 \text{ N}$$

ans : $\boxed{370\hat{i} + 900\hat{j} \text{ N}}$