

Final Exam Review Sheet - Solutions

- ① Since the collision is inelastic, you can't use conservation of energy. But, momentum is conserved in any collision. Remember to treat it as a vector:

$$(P_y)_{\text{before}} = (P_y)_{\text{after}}$$

$$0 = (.01 \text{ kg})(50 \text{ m/s}) \sin 28^\circ - (.0071 \text{ kg})(53.7 \text{ m/s}) \sin \theta$$

$$0 = .2347 - .3813 \sin \theta$$

$$\sin \theta = \frac{.2347}{.3813} = .6157 \Rightarrow \boxed{\theta = 38.0^\circ} \text{ ANS}$$

(Or, you could do it using x components instead.)

- ② A collision again. Since you're looking for angular speed, use angular momentum:

$$(I\omega_i)_{\text{DISK}} = (I\omega_f)_{\text{DISK}} + (I\omega_f)_{\text{HOOP}}$$

$$\left(\frac{1}{2}MR^2\right)\left(4 \frac{\text{RAD}}{\text{s}}\right) = \left(\frac{1}{2}MR^2 + MR^2\right)\omega_f$$

$$2MR^2 = \frac{3}{2}MR^2\omega_f$$

$$\frac{4}{3} = \omega_f$$

$$\boxed{\omega_f = 1.33 \frac{\text{RAD}}{\text{s}}} \text{ (A)}$$

$$\begin{aligned} \text{B) } \frac{KE_f}{KE_i} \times 100\% &= \frac{\frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_i\omega_i^2} \times 100\% = \frac{\left(\frac{3}{2}MR^2\right)\left(\frac{4}{3} \frac{\text{RAD}}{\text{s}}\right)^2}{\left(\frac{1}{2}MR^2\right)\left(4 \frac{\text{RAD}}{\text{s}}\right)^2} \times 100\% \\ &= \frac{8/3}{8} \times 100\% = \frac{1}{3} \times 100\% = \boxed{33.3\%} \end{aligned}$$

- ③ Read from graph: $A = 3 \mu\text{m}$ and $T = 4 \text{ ms} = .004 \text{ s}$

$$\text{So, } f = \frac{1}{T} = \frac{1}{.004 \text{ s}} = 250 \text{ Hz}$$

$$\text{and } \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{250 \text{ Hz}} = 1.372 \text{ m} \quad \begin{array}{l} \text{speed of sound, from table} \\ \text{in formula sheet.} \end{array}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.372} = 4.58 \text{ m}^{-1}$$

$$\omega = 2\pi f = 2\pi(250) = 1.57 \times 10^3 \text{ s}^{-1}$$

Harmonic wave going right: $y = A \sin(kx - \omega t + \phi)$

$$\text{ANS: } \boxed{y = (3 \mu\text{m}) \sin[(4.58 \text{ m}^{-1})x - (1.57 \times 10^3 \text{ s}^{-1})t]}$$

④ X components:

$$\Delta x = v_{ix} t$$

$$150 \text{ m} = (v_i \cos 40^\circ) t$$

$$t = \frac{150}{v_i \cos 40^\circ} = \frac{195.81}{v_i}$$

Y components:

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$100 = (v_i \sin 40^\circ) \left(\frac{195.81}{v_i} \right) + \frac{1}{2} (-9.8) \left(\frac{195.81}{v_i} \right)^2$$

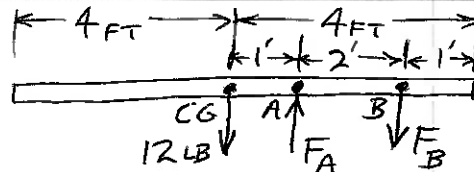
$$100 = 125.86 - \frac{187876}{v_i^2}$$

$$\frac{187876}{v_i^2} = 25.86$$

$$\sqrt{\frac{187876}{25.86}} = v_i \Rightarrow \boxed{85.2 \text{ m/s}}$$

⑤ Forces on board:

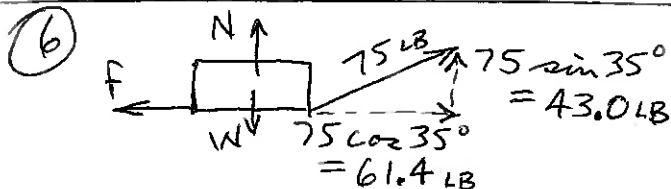
Use B as the pivot point, because that way, the unknown force we don't care about will disappear from the equation:



$$\sum \tau_B = 0$$

$$(0)(F_B) - (2 \text{ ft})(F_A) + (3 \text{ ft})(12 \text{ lb}) = 0$$

$$2F_A = 36 \Rightarrow \boxed{F_A = 18 \text{ lb}}$$



$$\sum F_y = 0$$

$$N + 43 - W = 0$$

$$\boxed{N = W - 43 \text{ lb}}$$

To move it, the 61.4 lb must be at least a tiny bit more than f_{max} , the maximum static friction.

$$f_{\text{max}} < 61.4 \text{ lb}$$

$$\mu_s N < 61.4$$

$$(.45)(W - 43) < 61.4$$

$$.45W - 19.4 < 61.4$$

$$.45W < 80.8$$

$$W < \boxed{180 \text{ lb}} \text{ ANS}$$

approximately 180 lb (actually a tiny bit less) is the most it can weigh.

⑦ $1 \Rightarrow$ where it enters, $2 \Rightarrow$ in basement.

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi (1.5 \text{ m})^2}{\pi (.25 \text{ m})^2} (.15 \text{ m/s}) = 5.40 \text{ m/s}$$

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$\cancel{40^{18} \text{ N}^2} + \frac{1}{2} (1000 \text{ kg/m}^3) (.15 \text{ m/s})^2 + 0 = \cancel{40^{18} \text{ N}^2} + \frac{1}{2} (1000 \text{ kg/m}^3) (5.4 \text{ m/s})^2 + (1000)(9.8) h_2$$

Take $h=0$ level to be at point 1.

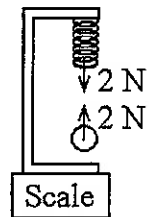
$$11.25 = 14580 + 9800 h_2$$

$$-14569 = 9800 h_2 \Rightarrow h_2 = \frac{-14569}{9800} = -1.49 \text{ m}$$

ANS: 1.49 m lower.

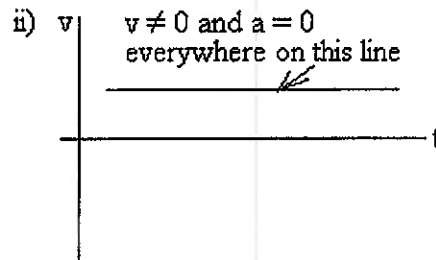
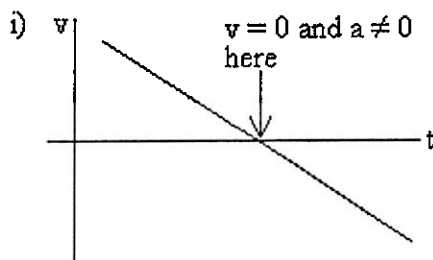
8. a. Convection.

b. 7 N, 7 N. In each case, the ball makes a 2 N downward force on the magnet & support assembly. In the picture on the left, the 2 N is because the ball is sitting on it. In the picture on the right, it's because of Newton's 3rd law as shown here. In part (i) of the question, it's half and half.



c. Same. Equal speeds means equal kinetic energies. Equal heights means equal potential energies. Direction of motion doesn't matter; energy is a scalar.

d. The slope is the acceleration. So, the first graph should have a point where $v = 0$, but the graph is not horizontal. The second graph should have a point where v is not 0, but the graph is horizontal. For example:



e. The +z direction. (Pointing out of the page.) Like other vectors describing rotation, angular momentum points along the axis of rotation. If the fingers of your right hand go around the orbit pointing in the direction of motion, your thumb points in the direction of L .