Phy 131 - Assignment 10

A.

Compare $x = (5\text{cm})\cos(2t + \frac{\pi}{6})$
to $x = A\cos(\omega t + \phi) \Rightarrow A = 5\text{cm},\ \omega = 2\frac{\text{RAD}}{\text{s}}$

so, $v_{MAX} = \omega A = (2\frac{\text{RAD}}{\text{s}})(5\text{cm}) = 10\frac{\text{cm}}{\text{s}}$ and $a_{MAX} = \omega^2 A = (2^2)(5) = 20\frac{\text{cm}}{\text{s}^2}$

A) $x = 5\cos[2(0) + \pi/6] = 5\underbrace{\cos(\pi/6\ \text{RAD})}_{.8660} \doteq \boxed{4.33\text{cm}}$

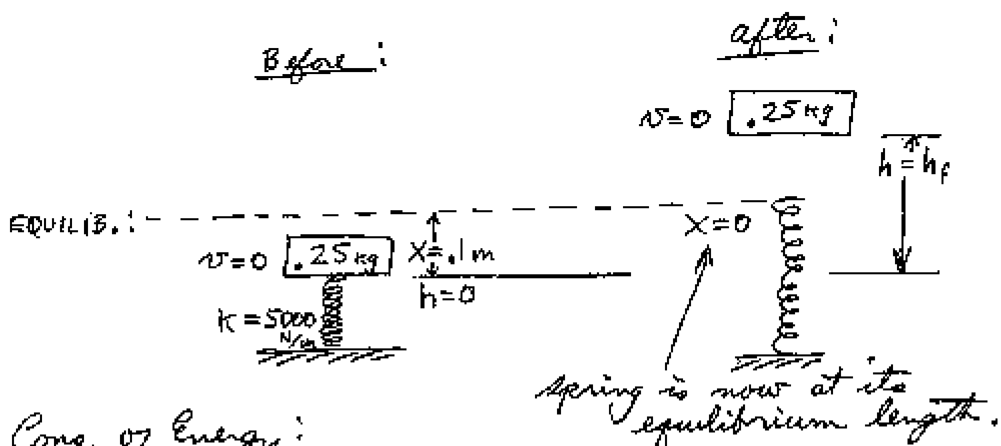
B) $v = -v_{MAX}\sin(\omega t + \phi) = -(10\frac{\text{cm}}{\text{s}})\sin(\frac{\pi}{6}\text{RAD}) = \boxed{-5.00\frac{\text{cm}}{\text{s}}}$

C) $a = -a_{MAX}\cos(\omega t + \phi) = -(20\frac{\text{cm}}{\text{s}^2})\cos(\frac{\pi}{6}\text{RAD}) = \boxed{-17.3\frac{\text{cm}}{\text{s}^2}}$

D) $\omega = 2$
$2\pi f = 2$
$f = \frac{2}{2\pi}$ → $f = \frac{1}{\pi}$
$T = \frac{1}{f} = \boxed{\pi\ \text{sec}}$

E) $\boxed{A = 5\text{cm}}$ as mentioned above.

B. 1. <u>More than</u>. From Hooke's law, k = F/x. The question says that with F the same, x is half as much. So, k is twice as much.



Cons. of Energy:

$E_i = E_f$

$(U_g + U_s + KE)_i = (U_g + U_s + KE)_f$

$0 + \frac{1}{2}(5000\frac{N}{m})(.1)^2 + 0 = (.25)(9.8)(h_f) + 0 + 0$

$25.0 = 2.45 h_f$

$h_f = \frac{25.0}{2.45} = \boxed{10.2\text{m}}$ ANS

C.

$$\omega = \sqrt{\frac{g}{\ell}} \quad \text{and} \quad \omega = 2\pi f \quad \text{so} \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{\ell}{g}}$$

a) 2, π and ℓ are all constants. T is a function of g. Rewriting this in a form which is easier to differentiate:

$$T = (2\pi\sqrt{\ell})\, g^{-\frac{1}{2}}$$

Differentiating: $dT = (2\pi\sqrt{\ell})(-\tfrac{1}{2} g^{-\frac{3}{2}}\, dg) = -\pi\sqrt{\frac{\ell}{g^3}}\, dg$

Don't forget the dg, from the chain rule. dg is the difference in g between the two locations.

For best results, use the average value for g. However, it changes so little it wouldn't make much difference if you used 9.80 or 9.79.

$$dT = -\pi\sqrt{\frac{2.50}{9.795^3}}\,(9.79 - 9.80) = \boxed{.00162\ \text{s}}$$

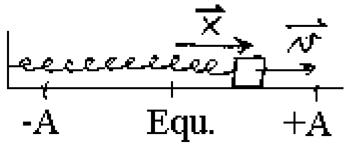
b) $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T_1 = 2\pi\sqrt{\frac{2.50}{9.80}} = 3.173488\ \text{s}$$

$$T_2 = 2\pi\sqrt{\frac{2.50}{9.79}} = 3.175108\ \text{s}$$

$$\Delta T = \boxed{.00162\ \text{s}}$$

D. 1. a. Yes. For example:



b. Yes. $\vec{a}$ always points in the direction of $\vec{F}$ (by $\vec{F} = m\vec{a}$), which is toward equilibrium. Sometimes it's also moving toward equilibrium.

c. No. Hooke's law, $\vec{F} = -k\vec{x}$, says $\vec{F}$ and $\vec{x}$ always have opposite directions. The direction of $\vec{F}$ is the direction of $\vec{a}$.

2. Energy at equilibrium = Energy at 15 cm

$$(\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2)_{eq} = (\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2)_{15cm}$$

$$\tfrac{1}{2}(.13\text{ kg})(5\text{ m/s})^2 + \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}(.13\text{ kg})v^2 + \tfrac{1}{2}(67\text{ N/m})(.15\text{ m})^2$$

$$1.625 = .065\, v^2 + .75375$$

$$.87125 = .065\, v^2$$

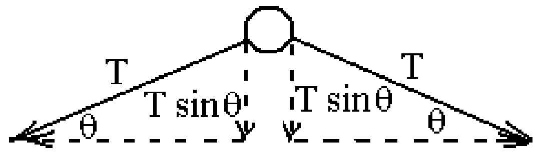
$$v = \sqrt{\frac{.87125}{.065}} = \boxed{3.66\text{ m/s}}$$

E. 1. Friction gradually removes its energy. (Air friction and a little internal friction in the spring as it flexes back and forth.)

2.

a. To show this is true, you should mention the following:

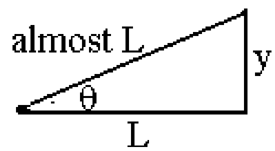
- The downward component from each rubber band is $T \sin\theta$



- The restoring force is the net downward force from the rubber bands:

$$-T\sin\theta - T\sin\theta = -2T\sin\theta$$

- since θ is small, $\sin\theta \approx y/L$



- So, $-2T\sin\theta \approx -2T\,(y/L) = \boxed{-(2T/L)y}$

(Like the pendulum I discussed in class, this system is harmonic only for small displacements.)

b. - comparing the result from part a, $F = -(2T/L)y$, to Hooke's Law, $F = -ky$, gives $k = 2T/L$.

- The angular frequency of any Harmonic oscillator is $\omega = \sqrt{\frac{k}{m}}$.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T/L}{m}} = \boxed{\sqrt{\frac{2T}{mL}}}$$

F. h = 0 throughout the process, so $U_g = mgh$ can be ignored. x = amount the spring gets compressed, so x is what we're looking for.

$$E_i + W = E_f$$

$$\tfrac{1}{2} m v^2 + (-fx) = \tfrac{1}{2} k x^2$$

Spring not compressed initially, so $\frac{1}{2}kx^2 = 0$

Work done by friction

Block momentarily at rest so $\frac{1}{2}mv^2 = 0$

$$\tfrac{1}{2}(3.0\,\text{kg})(1.5\,\text{m/s})^2 - (\mu n)x = \tfrac{1}{2}(30\,\text{N/m})x^2$$

$f = \mu n$

n = mg in this case.

mg = (3 kg)(9.8 m/s²) = 29.4 N

$$3.375 - \underbrace{(.3)(29.4\,\text{N})}_{8.82}x = 15x^2$$

$$\underbrace{15}_{a}x^2 + \underbrace{8.82}_{b}x \underbrace{- 3.375}_{c} = 0$$

quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-8.82 \pm \sqrt{8.82^2 - 4(15)(-3.375)}}{2(15)}$$

$$= \frac{-8.82 \pm \sqrt{77.8 + 202.5}}{30}$$

$$= \frac{-8.82 \pm 16.74}{30}$$

using (+) ↘ using (−) ↘

$$= .264\,\text{m} \quad \text{or} \quad -.852\,\text{m}$$

Which root is physically meaningful? When I wrote W = – fx, I was using x for the magnitude of the displacement. A magnitude is a positive number.

Answer: .264 m