# Overview of Chapter 1

You should have mastered most of the material in the previous algebra course. Here is a brief overview of some of the topics you should be familiar with, along with a few concepts that may be new.

## 1. Sets, Union, Intersection and Interval Notation

***Set*** - a collection of elements

***Union*** - all elements in either of the sets (or both) - denoted: $\cup$

***Intersection*** - all elements present in both sets - denoted: $\cap$

***Empty Set*** - a set with no elements, denoted: { } or $\varnothing$

***Roster Notation*** - a method of expressing a set as a list of elements

***Set-Builder Notation*** - a method of expressing a set of numbers using equalities and inequalities

***Interval Notation*** - a method of expressing a set of real numbers from left endpoint to right endpoint

Example - Define two sets using roster notation and compute the union and intersection:

1. A = {2, 3, 4} and B = {3, 4, 5}

$A \cup B$ = {2, 3, 4, 5}

$A \cap B$ = {3, 4 }

2. A = {2, 4, 6} and B = {3, 5, 7 }

$A \cup B$ = {2, 3, 4, 5, 6, 7}

$A \cap B$ = { } OR $\phi$

For infinitely large sets of real numbers, roster notation is impractical, so we use set-builder, graphing notation, or interval notation. When dealing with these sets, endpoints are either included as part of the set (inclusive), or excluded from the set (exclusive).

We use square brackets, [ or ], to show *inclusion*, and rounded brackets, ( or ), to show *exclusion*.

Examples

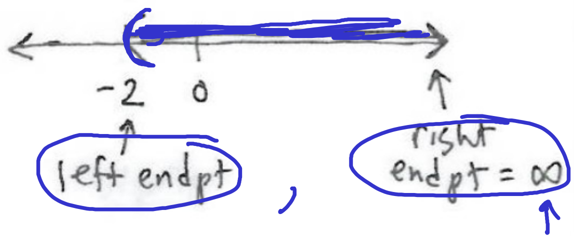
1. Set A is all numbers larger than -2. (This means that -2 is NOT included in the set.)

graph

interval

$A = \{ \underbrace{x > -2} \}$

-2 not included




,

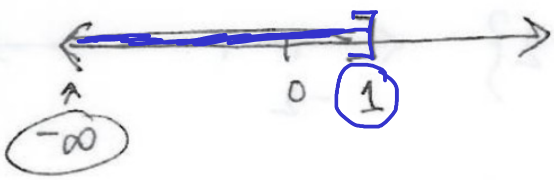
(-2, ∞)

∞ always round bracket

2. Set B is all numbers less than or equal to 1.

interval

$B = \{ x \leq 1 \}$

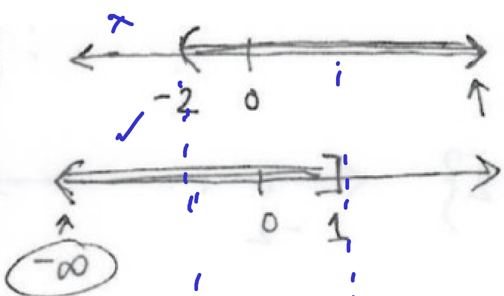



(-∞, 1]

Now, consider the union and intersection of these two sets.

$A = \{ x > -2 \}$

$B = \{ x \leq 1 \}$

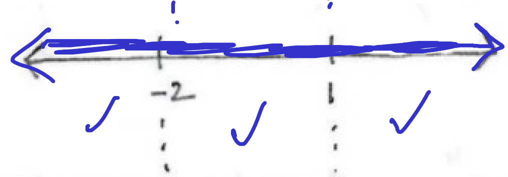



3 subintervals

A ∪ B

"either"

which regions are in at least one of the sets?

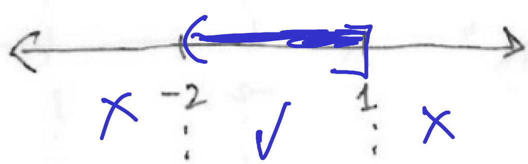



(-∞, ∞)

ℝ all real numbers

A ∩ B

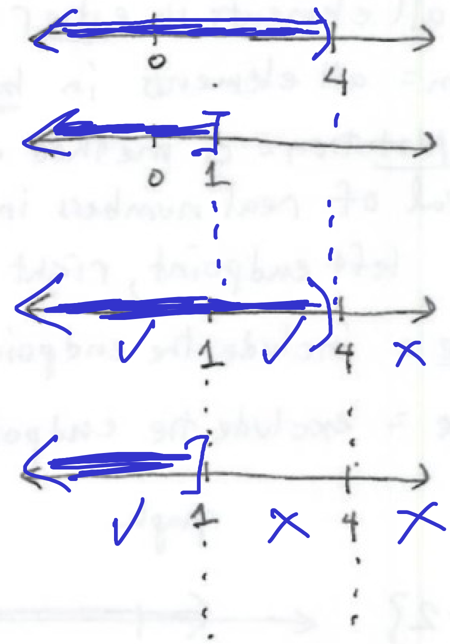
which regions are in both sets simultaneously?




[-2, 1]

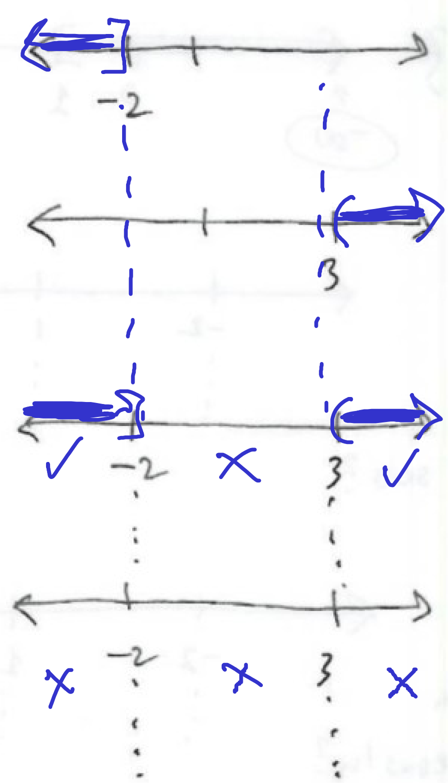
**Example 2**

| | graph | interval |
|---|---|---|
| $A = \{x < 4\}$ | | $(-\infty, 4)$ |
| $B = \{x \leq 1\}$ | | $(-\infty, 1]$ |
| $A \cup B$ "either" | | $(-\infty, 4)$ |
| $A \cap B$ "both" | | $(-\infty, 1]$ |




**Example 3**

| | graph | interval |
|---|---|---|
| $A = \{x \leq -2\}$ | | $(-\infty, -2]$ |
| $B = \{x > 3\}$ | | $(3, \infty)$ |
| $A \cup B$ "either" | | $(-\infty, -2] \cup (3, \infty)$ |
| $A \cap B$ "both" | | $\emptyset$ |



## Variable Expressions

***A. Evaluating variable expressions.*** Just a matter of plugging the current value of the variable into the expression, and simplifying.

Examples:

1. Evaluate $\sqrt{b^2 - 4ac}$ when $a$ = 2, $b$ = -7 and $c$ = -5. Hint: plug in negative numbers with parentheses!

$$\sqrt{(-7)^2 - 4(2)(-5)}$$

$$\sqrt{49 + 40}$$

$$= \sqrt{89} \approx 9.4$$

$$(-7)^2 = (-7)(-7) = 49$$

$$-4(2)(-5) = +40$$

$\sqrt{81}$ $\sqrt{89}$ $\sqrt{100}$

9 $\sim 9.4$ 10

2. Evaluate $\dfrac{2c^2 - ab}{-4a}$ using the same values for $a$, $b$ and $c$ above.

$a = 2, \ b = -7, \ c = -5$

$$\frac{2(-5)^2 - 2(-7)}{-4(2)} = \frac{2 \cdot 25 - 2(-7)}{-8} = \frac{50 + 14}{-8}$$

$$= \frac{+64}{-8} = -8$$

Order of operation

1. Parentheses
2. Exponents
3. M/D
4. A/S

***B. Simplifying Variable Expressions.*** Basic guideline:

1. Combine like terms
2. Distribute
3. Work on parentheses first, from innermost to outermost.

Simplify the following expressions:

1. $5(2a-3b)-6(a+4b)$

distribute 5, then -6

$10a-15b-6a-24b$

combine

$4a-39b$

2. $3-4[5y+2(3-7x)]$

dist. +2

$3-4[5y+6-14x]$

dist -4

$3-20y-24+56x = -21-20y+56x = 56x-20y-21$

***C. Solving First Degree Equations.*** Basic guideline:

- Simplify each side (combine terms, distribute, etc.)
- Add/subtract the variable terms - move to one side.
- Add/subtract the constant terms - move to the other side.
- Divide each side by the coefficient of the variable.

$\cancel{x^2}$ $\cancel{\sqrt{\ }}$ $\cancel{\frac{1}{x}}$

$3x^1+2=7$

There are three possible scenarios when solving first-degree (also known as linear) equations.

***1.*** You get a single solution. This is the typical result. $x=2$, $x=^-3.7$

***2.*** Every possible choice of x is a solution. This is called an identity, because it's always true.

***3.*** Every possible choice of x is not a solution. This is called a contradiction, because it's never true.

Examples - Solve the following equations for x:

1. $2(3x-7) = 3x + 8x - 1$

① simplify
② x-terms to one side
③ constants other
④ divide coefficient

$\cancel{6x} - 14 = 11x - \cancel{1}$
$-\cancel{6x} \quad +1 \quad -6x \quad +\cancel{1}$
$-13 = 5x$

$\frac{5x}{5} = \frac{-13}{5}$

$x = \frac{-13}{5}$ ✓ or $-2.6$

2. $7x+21 = 7(x+3)$

$\cancel{7x} + 21 = \cancel{7x} + 21$ identity
$-\cancel{7x} \quad -\cancel{7x}$

$21 = 21$ all x terms drop out!

If the **variable completely drops out**,
and the resulting equation is **true**, the original equation is an identity. all values of x work.

3. $5x-2 = 5(x-2)$

$\cancel{5x} - 2 = \cancel{5x} - 10$
$-\cancel{5x} \quad -\cancel{5x}$

$-2 = -10$ not true!

If the **variable completely drops out**,
and the resulting equation is **false**, the original equation is a contradiction.

*If your equation contains fractions, you may be able to simplify the equation first by clearing out denominators. Accomplish this by multiplying both sides of the equation by the Least Common Multiple (LCM) of the denominators.*

4. Solve: $\frac{3}{4}x = \frac{2}{3}$ The LCM of the denominators 3 and 4 is 12.

$$\frac{\cancel{12}^{3}}{1} \cdot \frac{3}{\cancel{4}}x = \frac{\cancel{12}^{4}}{1} \cdot \frac{2}{\cancel{3}}$$

$$9x = 8$$

$$\frac{9x}{9} = \frac{8}{9}$$

$$x = \frac{8}{9}$$

5. Solve: $10\left(\frac{2}{5} - \frac{x+1}{10} = 3\right)$. The LCM of the denominators 5 and 10 is 10.

$$\frac{\cancel{10}^{2}}{1} \cdot \frac{2}{\cancel{5}} - \frac{\cancel{10}^{1}}{1} \cdot \frac{(x+1)}{\cancel{10}} = \frac{10}{1} \cdot \frac{3}{1}$$

$$4 - 1(x+1) = 30$$

$$4 - 1x - 1 = 30$$

$$3 - 1x = 30$$

$$-3 \qquad -3$$

$$-1x = 27$$

$$\frac{-1x}{-1} = \frac{27}{-1}$$

$$x = -27$$

check your work.

***Literal Equations - Equations that contain more than one variable.***

Consider the formula to compute the perimeter of a rectangle: P = 2L + 2W.

This is a literal equation, and it is already solved for P. We can solve this equation for either of the other variables:

$\boxed{W}$

Ex1 - Solve P = 2L + 2W for **W**.

The trick: consider all terms that contain a W as variable terms, and all terms that don't as constant terms.

$$P = 2L + 2W$$

constant terms    variable term

Step 1 - Simplify each side - already done.

Step 2 - Move all variable terms to one side - already done, 2W is the only variable term, on right.

Step 3 - Move all constant terms to the other side.

$$P = 2L + 2W \qquad \text{subtract 2L from both sides.}$$

$$-2L \quad -2L$$

$$P - 2L = 2W$$

Step 4 - Divide by the coefficient of W.

$$\frac{P-2L}{2} = \frac{2W}{2} \qquad w = \frac{P-2L}{2} \quad \ldots \quad \frac{P}{2} - \frac{2L}{2} = \frac{P}{2} - L$$

You try it ... solve the resulting equation for L.

$$\frac{2}{1} \cdot \frac{W}{1} = \frac{(P-2L)}{\cancel{2}} \cdot \cancel{\frac{2}{1}} \qquad \text{for } L \qquad LCD = 2$$

$$2w = P - 2L$$

$$-P \qquad -P$$

$$\frac{2w - P}{-2} = \frac{-2L}{-2} \qquad \sim\sim \text{div by} \sim\sim \; -2 \qquad L = \frac{2w-P}{-2}$$

Ex2 - Solve for y: $3xy - 5x = 2(3x - 7xy) + 10x$

solve for y

$6x - 14xy + 10x$

$3xy - \cancel{5x} = 16x - \cancel{14xy}$

$+14xy \quad +\cancel{5x} \quad | \quad +5x \quad +\cancel{14xy}$

$\dfrac{17xy}{17x} = \dfrac{21x}{17x}$

div by coeff of y

$y = \dfrac{21\cancel{x}}{17\cancel{x}} = \dfrac{21}{17}$

Ex3 - Solve this equation for C: $F = \dfrac{9}{5}C + 32$

LCD = 5

$5 \cdot F = \dfrac{\cancel{5}}{1} \cdot \dfrac{9}{\cancel{5}} C + 5 \cdot 32$

$5F = 9C + \cancel{160}$

$-160 \qquad -\cancel{160}$

$\dfrac{5F - 160}{9} = \dfrac{9C}{9}$

$C = \dfrac{5F - 160}{9}$