Riemann Sum Lab – Fall 2005

For each function below,

1. State function and interval: For this example, it's \( f(x) = 3x^2 - x - 10 \) on \([-5, 4]\).
2. Print out the Riemann Sum for the interval and partition you chose.
3. Plot a graph of \( f(x) \) with superimposed rectangles on the appropriate interval \([-5, 4]\). These rectangles should accurately reflect the rectangles associated with your choice of a partition.

4. Show all work for integral of \( f(x) \) over the interval.
   \[
   \int_{-5}^{4} (3x^2 - x - 10) \, dx
   \]
   and compare results with Riemann sum. Remember, Riemann sums often provide an inaccurate portrayal of the actual value of the integral. Do not be concerned if these values do not mesh.

You should do the above work for the three functions below. You get to choose the interval and the number of subintervals. You must choose more than three subintervals for at least one of the functions. As always, cut and paste so all parts for each problem appear on a single sheet of paper! Each group should consist of two or fewer students. Groups must do work independently from other groups. Good luck.

Functions:

1) \( f(x) = 3x^2 + 4x - 12 \)

2) \( f(x) = 3 \sin(x) + \cos(x) \) (remember: RADIANS MODE!!!)

3) \( f(x) = x \sqrt{x^2 + 16} \)
This is EXACTLY what each page should look like when you have finished with your lab. Notice that you superimpose the rectangles (by hand) on the printout of the graph. On each rectangle, label the coordinates of the point that marks the height of the rectangle. Use scissors and tape to cut and paste (literally) the graph onto the printout of the Riemann Sum.

**Riemann Sum lab for Joe Student**

Function is: $3x^2 - x - 10$

Interval is $[-5, 4]$

Number of subintervals is 3

<table>
<thead>
<tr>
<th>Interval</th>
<th>$\Delta x$</th>
<th>$w_i$</th>
<th>$f(w_i)$</th>
<th>$f(w_i) \Delta x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[-5, 0]$</td>
<td>5.000</td>
<td>-2.0000</td>
<td>4.0000</td>
<td>20.0000</td>
</tr>
<tr>
<td>$[0, 3]$</td>
<td>3.000</td>
<td>1.0000</td>
<td>-8.0000</td>
<td>-24.0000</td>
</tr>
<tr>
<td>$[3, 4]$</td>
<td>1.000</td>
<td>3.0000</td>
<td>14.0000</td>
<td>14.0000</td>
</tr>
</tbody>
</table>

Riemann Sum equals: 10

**Definite Integral**

\[
\int_{-5}^{4} (3x^2 - x - 10) \, dx = \]

\[
\left[ x^3 - \frac{x^2}{2} - 10x \right]_{-5}^{4} =
\]

\[
(4^3 - \frac{4^2}{2} - 10(4)) - \left( (-5)^3 - \frac{(-5)^2}{2} - 10(-5) \right) = (64 - 8 - 40) - (-125 - 12.5 + 50) \]

\[
16 - (-87.5) = 103.5
\]