Newton's Method
Lab Assignment due end of the day, Tuesday December 6th.

1. To solve the equation f(x) = g(x), let h(x) = f(x) - g(x). Finding zeros of h(x) is equivalent to finding solutions to the equation f(x) = g(x).

2. Guess a first approximation \( x_0 \) for a root of the equation h(x). A graph of h(x) may help.

3. Use the first approximation to get a second, the second to get a third, and so on, using the iterative formula:
\[
x_{n+1} = x_n - \frac{h(x_n)}{h'(x_n)}
\]
where \( h'(x_n) \neq 0 \).

Using Maxima to apply Newton's method:
1. Define a function \( H(x) \) equal to the original function, a function \( HP(x) \) equal to the derivative of \( H(x) \), and a function \( N(x) := x - H(x)/HP(x) \). Define a function \( H(x) \) by using the \( H(x) := \) format.

2. Evaluate \( N(x) \) at your initial approximation \( x_0 \) for a root. \( N(x) \) will return the next approximation \( x_1 \). Simply plug this second approximation back into \( N(x) \) to get a third approximation, a fourth, and so on.

3. Stop when your approximation is good to 6 decimal places, namely when \( N(x) \) doesn't change from one approximation to the next for the first six decimal places:

Here's an example: Solve \( x^3 = \sin(2x) \). Let \( H(x) := x^3 - \sin(2x) \), \( HP(x) := 3x^2 - 2\cos(2x) \) and \( N(x) := x - H(x)/HP(x) \). Use a colon in front of the equal sign to "assign" functions to their equations, and use * for multiplication:

Maxima
(C1) \( H(x) := x^3 - \sin(2x) \)
(D1) \[
3
H(x) := x - \text{SIN}(2\, x)
\]

(C2) \( HP(x) := 3x^2 - 2\cos(2x) \)
(D2) \[
2
HP(x) := 3\, x - 2\, \text{COS}(2\, x)
\]

(C3) \( N(x) := x - H(x)/HP(x) \)
(D3) \[
N(x) := x - \frac{H(x)}{HP(x)}
\]

(In the next step, notice that we use \( N(2.0) \) and not \( N(2) \) for our initial guess. This allows maxima to return the result in decimal form. Otherwise, you'll get your results in fraction form.)

(C4) \( N(2.0) \)
(D4) \[1.34195434905399\]
Notice, the last two results didn't change in the first seven decimal places. This means we can stop, since we need the result to be good to at least six places.

Try this with three problems: (1) Find two, distinct zeros (by using two different initial values $x_0$) of the function: $f(x) = x^3 - 6x^2 - x + k$, where $0 < k < 30$, (2) solve the equation of the form: $2\sin(x) = x^3 + k$, where $k \neq 0$ and (3) find an approximation to $\sqrt[5]{k}$ for some value of k (note: approximating $\sqrt[5]{k}$ is the same as solving $x^5 - k = 0$, isn't it??).

Remember to solve the first equation twice by use a different initial choices for $x_0$.

Make sure at the top of each printout, you CLEARLY STATE the EQUATION or PROBLEM you were ORIGINALLY solving. On each printout, specifically label each successive approximation with $x_1, x_2, x_3, \ldots$ and so on.

Good luck!