PHY 133 homework, continued. (Reading refers to Serway \& Jewett's Physics for Scientists \& Engineers with Modern Physics, ${ }^{\text {th }}$ ed., "Hybrid Edition.")

## Sec. 5 - Special Relativity

Read: Ch. 39: sec 1-8, except Relativistic Doppler Effect p. 949-50.
A. 1. ( 2 points) The speed of an electron is calculated from the voltage which accelerated it, using the formula $\mathrm{KE}=1 / 2 \mathrm{mv}^{2}$. In each case, state whether the answer is correct to at least three significant figures:
a. What if the answer is $5.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ ?
b. What if the answer is $1.00 \times 10^{8} \mathrm{~m} / \mathrm{s}$ ? $\qquad$
c. What if the answer is $5.00 \times 10^{7} \mathrm{~m} / \mathrm{s}$ ? $\qquad$
d. What if the answer is $1.00 \times 10^{7} \mathrm{~m} / \mathrm{s}$ ? $\qquad$
2. (8) An observer on Earth sees spacecraft A approaching Earth at .500c. An observer on spacecraft A sees spacecraft B approaching him from behind at .600 c . What does the observer on Earth see as the speed of $B$ ?
ans: .846c
B. 1. (2pts) A certain electron's total energy is 1.17 MeV . What is its kinetic energy?
ans: .66 MeV
2. (8) A spaceship travels at .750 c relative to Earth. If the spaceship fires a small rocket in the forward direction, how fast relative to the ship must it be fired to travel at .950 c relative to Earth?
ans: .696c
C. 1. (2 pts) Explain in words why it is impossible for an object (of nonzero mass) to reach a speed of c , regardless of the size and duration of the force on it.
8. (8) Consider the decay ${ }^{55} \mathrm{Cr} \rightarrow{ }_{25}{ }_{25} \mathrm{Mn}+\mathrm{e}$, where e is an electron. The ${ }^{55} \mathrm{Cr}$ nucleus has a mass of 54.9279 u , and the ${ }^{55} \mathrm{Mn}$ nucleus has a mass of 54.9244 u . (a) Calculate the mass difference between the two nuclei in electron volts. (b) What is the maximum kinetic energy of the emitted electron?
ans: $3.26 \mathrm{MeV}, 2.75 \mathrm{MeV}$
D. 1. (3 pts) An astronaut moves away from the earth at a speed close to the speed of light. What changes, if any, would be measured in the astronaut's size and pulse rate by (a) an observer on Earth? (b) the astronaut?
2. Consider a proton with an energy of 2.00 joules. (Cosmic ray particles have been seen with this much or even a little more.) Assume this proton has been moving with a constant velocity since leaving the center of the Milky Way galaxy, $2.50 \times 10^{17} \mathrm{~km}$ away as measured from the Earth's frame of reference. In the proton's reference frame,
a. What is the distance from the galactic center to the Earth?
b. How fast is the Earth moving toward the proton? (A lengthy calculation is not necessary - just think about what your value for $\gamma$ means.)
c. How much time does the trip take?
ans: $1.88 \times 10^{7} \mathrm{~km}, \mathrm{c}, 62.7 \mathrm{~s}$
E. 1. (2 pts) What two speed measurements do two observers in relative motion always agree on?
2. (8) The mass of a $\pi^{-}$meson is $2.49 \times 10^{-28} \mathrm{~kg}$ and its average lifetime in its own frame of reference is $2.6 \times 10^{-8} \mathrm{~s}$. If the pion moves at .95 c relative to an observer on Earth, what will that observer measure for (a) its average lifetime, (b) the average distance it travels before decaying? (c) its momentum?
ans: $83.3 \mathrm{~ns}, 23.7 \mathrm{~m}, 2.27 \times 10^{-19} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$
F. A pion at rest $\left(\mathrm{m}_{\pi}=2.4881 \times 10^{-28} \mathrm{~kg}\right)$ decays into a muon $\left(\mathrm{m}_{\mu}=1.8835 \times 10^{-28} \mathrm{~kg}\right)$ and an antineutrino ( $\mathrm{m}_{\bar{v}} \approx 0$.) The reaction is written $\pi^{-} \rightarrow \mu^{-}+\bar{v}$. It can be shown from conservation of momentum that $12.15 \%$ of the energy produced by this decay goes to the kinetic energy of the muon and the rest goes to the antineutrino. Find (a) the kinetic energy of the muon. (b) $\gamma$ for the muon. ans: $6.60 \times 10^{-13} \mathrm{~J}, 1.04$

## Sec. 6 - Wave-Particle Duality

Read: Ch 40: start $-\sec 2, \sec 4,5 \& 8 ;$ Ch. 41: first part of $\sec 5$, sec. 6
A. 1. (2 pts) Some stars look red, and some look blue. Which has the higher surface temperature? Explain.
2. (8) Molybdenum has a work function of 4.20 eV .
(a) Find the cutoff frequency and cutoff wavelength for the photoelectric effect.
(b) Calculate the stopping potential if the light has a wavelength of 180 nm .
ans: $1.02 \mathrm{PHz}, 295 \mathrm{~nm}, 2.69 \mathrm{~V}$
B. 1. ( 2 pts ) $\mathrm{A}^{3} \mathrm{He}$ nucleus and a ${ }^{4} \mathrm{He}$ nucleus, each with 4 MeV of kinetic energy, approach a thin barrier 5 MeV high. Which, if either, is more likely to tunnel through? (For $1 / 2 \mathrm{mv}^{2}$ to be the same, the lighter ${ }^{3} \mathrm{He}$ is going faster.)
2. (8) Show that the de Broglie wavelength of an electron accelerated from rest through a potential difference V is given by $\lambda=\frac{\text { some constant }}{\sqrt{V}}$. What is the value of the constant if V is in volts and $\lambda$ is in nanometers? (Assume its speed is non-relativistic.)
ans: 1.226
C. The wave nature of particles was first verified in 1927 when Davisson and Germer, at Bell Labs, demonstrated diffraction of electrons from a crystal of nickel. Somewhat simplified data from one of their trials is given below:
distance between atomic planes: . $81 \AA$
accelerating voltage: 65 volts
a. Find the electrons' speed from the accelerating voltage.
b. Put the electrons' de Broglie wavelength into the Bragg equation to predict the angle where the first order maximum would occur if de Broglie were correct.
ans: $4.78 \times 10^{6} \mathrm{~m} / \mathrm{s}, 70^{\circ}\left(\right.$ Observed value $=68^{\circ}-$ not a bad match. $)$
D. Derive the Stefan-Boltzmann law from the Planck radiation law, as follows: The total power per unit area radiated by a blackbody at a temperature T is $\int_{0}{ }^{\infty} \mathrm{I}(\lambda, \mathrm{T}) \mathrm{d} \lambda$, where $\mathrm{I}(\lambda, \mathrm{T})$ is given by the Planck radiation law. Show that this is equal to $\sigma \mathrm{T}^{4}$, where $\sigma$ is a constant.

Hints:

- Make the change of variable $x=(h c) /(\lambda k T)$.
- Treat T as a constant in performing the integral.
- Use the fact that $\int_{0}^{\infty}\left(\mathrm{x}^{3} \mathrm{dx}\right) /\left(\mathrm{e}^{\mathrm{x}}-1\right)=\pi^{4} / 15$

Your solution should show what $\sigma$ is in terms of $\mathrm{h}, \mathrm{c}, \mathrm{k}$ and $\pi$.
ans: $\sigma=\left(2 \pi^{5} \mathrm{k}^{4}\right) /\left(15 \mathrm{c}^{2} \mathrm{~h}^{3}\right)$
E. 1. (2 pts) If matter has a wave nature, why is this not observable in our daily experiences?
2. (8) Let's say that Planck's radiation law was $I=\lambda^{-3} e^{-1 /\left(\lambda T^{2}\right)}$ instead of the equation on the formula sheet. In this alternate universe, what would be the relationship between $\lambda_{\max }$ and T that would replace Wein's displacement law? ( $\lambda_{\max }$ is the wavelength where I is a maximum. Treat T as a constant.)

$$
\text { ans: } \lambda \mathrm{T}^{2}=\frac{1}{3}
$$

F. Estimate the width of the $\mathrm{n}=2$ to $\mathrm{n}=1$ line in the spectrum of hydrogen as follows. Background information: An electron in the $n=2$ state of hydrogen remains there an average of about 1.6 ns before dropping to the $\mathrm{n}=1$ state. It loses 10.2 eV as it falls. This 10.2 eV becomes the energy of a photon given off by the atom.
a. From the fact that it has this energy only 1.6 ns , what is the uncertainty in the electron's energy while in the $\mathrm{n}=2$ state? (This would be the same as the uncertainty in the emitted photon's energy.)
b. Write the relationship which gives a photon's wavelength, $\lambda$, as a function of its energy, E.
c. Take the differential of this relationship between $\lambda$ and $E$ to get a relationship between $\mathrm{d} \lambda$, the uncertainty in $\lambda$, and dE , the uncertainty in E .
d. Fill values into the expression from (c), including your $\Delta E$ from part (a). This is how far from the center of the line to either edge. Multiply by 2 to get its width.
ans: $4.11 \times 10^{-7} \mathrm{eV}, 9.80 \mathrm{fm}$

## Sec. 7 - Bohr Model of Hydrogen/X-Ray Spectra

Read: Ch. 42: $\sec 1-3,8 \& 9$.
A. The $K_{\alpha}$ line is the one emitted when an electron undergoes a transition from the $L$ shell $(n=2)$ to the $K$ shell $(\mathrm{n}=1)$. Calculate the frequency of the $\mathrm{K}_{\alpha} \mathrm{x}$-ray from a nickel target $(\mathrm{Z}=28)$.
ans: $1.80 \times 10^{18} \mathrm{~Hz}$
B. 1. (2 pts) Can the electron in the ground state of hydrogen absorb a photon of energy (a) 10.0 eV ? (b) 14.0 eV ?
2. (8) What value of n is associated with the 94.96 nm line in the Lyman hydrogen series?
ans: 5
C. 1. ( 2 pts ) Exciting an electron from the 1 s state to the 2 s state requires about twice as much energy for a $\mathrm{He}^{+}$ion as for a neutral helium atom. Explain why.
2. (8) A tungsten target is struck by electrons that have been accelerated from rest through a 40 kV potential difference. Find the shortest wavelength of the bremsstrahlung radiation emitted.
ans: .0310 nm
D. 1. ( 2 pts ) Suppose that the electron in a hydrogen atom obeyed classical mechanics rather than quantum mechanics. Why would such an atom emit a continuous spectrum rather than the observed line spectrum?
2. (8) In the Bohr model of hydrogen, what is the wavelength of the electron in
a. The ground state $(\mathrm{n}=1)$ ?
b. The first excited state ( $\mathrm{n}=2$ )?
ans: $3.32 \AA$ Á, 6.65 Á
E. A hydrogen atom is in its $\mathrm{n}=2$ state. According to the Bohr theory, this atom's radius is $2.117 \AA$. Using the Bohr theory, calculate
(a) the angular momentum of the electron,
(b) the linear momentum of the electron,
(c) the kinetic energy
(d) the total energy, $\mathrm{E}=\mathrm{KE}+\mathrm{U}$
ans: $2.11 \times 10^{-34} \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}, 9.97 \times 10^{-25} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}, 3.40 \mathrm{eV},-3.40 \mathrm{eV}$
F. Atom without a nucleus: PHY 132 will soon cover how an electron in a magnetic field can follow a circular path. Such orbits have energy levels (called Landau levels) similar to those in an atom. The force
on the electron is toward the center of its orbit and has a magnitude of $\mathrm{F}=\mathrm{qvB}$ where q is the charge, v is the speed, and $B$ is the magnetic field. Using this in place of Coulomb's law, repeat the procedure used for the Bohr atom in class to find the radius of the allowed orbits from the electron's deBroglie wavelength. Show how the answer follows from basic principles, not just the answer by itself. Express your answer in terms of $\mathrm{q}, \mathrm{B}, \hbar$ and n , where $\mathrm{n}=1,2,3 \ldots$ (Like the Bohr model of hydrogen, this approach to this system is not entirely accurate in all details.)

$$
\text { ans: } r=\sqrt{\frac{n \hbar}{q B}}
$$

## Sec. 8 - Wave Functions/The Square Well

Read: Ch. 40: sec 7; Ch. 41: sec. $1-3$.
A. 1. (2 points) The two lowest energy levels in hydrogen are -13.6 eV and -3.4 ev . Use the uncertainty principle to estimate how much time an electron can spend in a superposition of both states.
2. (8) A particle in an infinite square well of width L is in the ground state, $\mathrm{n}=1$. What is the probability of finding it the leftmost third of the well, $0 \leq x \leq L / 3$ ? (Notice the table of integrals in your formula handout.) ans: 1955
B. A nucleus is often approximated by a square well. Calculate the wavelength and energy of the photon emitted when a proton goes from $\mathrm{n}=2$ to the ground state in a square well of width 10 fm (a typical nuclear diameter.) ans: $202 \mathrm{fm}, 6.14 \mathrm{MeV}$
C. 1. ( 2 pts ) For a particle in a square well, in an excited state, the probability density is zero at certain points between the walls. Does this mean that the particle cannot cross these points? Explain.
2. (8) A particle with zero energy has a wave function given by $\psi(x)=A x e^{-x^{2}}$. Find the potential energy function, U , as a function of x .

$$
\text { ans: } \mathrm{U}=\left(\hbar^{2} / \mathrm{m}\right)\left(2 \mathrm{x}^{2}-3\right)
$$

D. The wave function for a particle in a one dimensional box of width $L$ is $\psi(x)=A \sin (n \pi x / L)$. Use the normalization condition (which in this case becomes $\int_{0}{ }^{L}|\psi|^{2} d x=1$ ) to find the constant, A, in terms of $L$ etc. (You must show a complete, step by step solution for full credit.)

$$
\text { ans: } \mathrm{A}=\sqrt{2 / L}
$$

E. 1. (2 pts) Use the uncertainty principle to explain why a particle in a confined space has a zero-point energy.
2. (8) For the $\mathrm{n}=4$ state of a particle in a one dimensional box of length L ,
(a) Sketch a graph of the probability density function from $x=0$ to $x=L$.
(b) Find the values of $x$ where the probability density is a maximum.
(c) Find the values of x where the probability density is a minimum.
ans: $\max : \mathrm{x}=\mathrm{L} / 8,3 \mathrm{~L} / 8,5 \mathrm{~L} / 8,7 \mathrm{~L} / 8 ; \min : \mathrm{x}=0, \mathrm{~L} / 4, \mathrm{~L} / 2,3 \mathrm{~L} / 4, \mathrm{~L}$
F. 1. (1 point) What is the maximum distance between two particles at which they can remain entangled?
2. (9 points) A particle is free to move between $x=0$ and $x=L$, but at those points it encounters infinitely hard walls. Given that the solution to $-\frac{\hbar^{2}}{2 \mathrm{~m}} \frac{\mathrm{~d}^{2} \psi}{\mathrm{dx}^{2}}=\mathrm{E} \psi$ is $\psi=\mathrm{A} \sin \left(\sqrt{\frac{2 \mathrm{mE}}{\hbar^{2}}} \mathrm{x}+\phi\right)$, where A and $\phi$ are constants, show how it follows from the boundary conditions that

- $\phi=0$ and
- the energy levels are given by the expression on your formula sheet.
(That is, be able to repeat what I showed you in class. Show a complete, step by step solution.)


## Sec. 9 - The 3-d Square Well/Hydrogen

Read: Ch 42: sec. $4-6$. Your text does not cover the 3 dimensional square well, so be sure you have complete notes on this.
A. 1. (2 points) According to Bohr's theory, what is the energy of a hydrogen atom in a 1 s state? According to the Schrodinger treatment, what is the energy of a hydrogen atom in a 1s state?
2. (8) Find all possible values for $L, L_{Z}$, and $\theta$ for an electron in a 3d state of hydrogen.
ans: $\mathrm{L}=\sqrt{6} \hbar ; \mathrm{L}_{\mathrm{Z}}=-2 \hbar,-\hbar, 0, \hbar$ or $2 \hbar ; \theta=145^{\circ}, 114^{\circ}, 90^{\circ}, 65.9^{\circ}$ or $35.3^{\circ}$
B. The radial part of the wave function for an electron in a certain state of hydrogen is $\psi(r)=A r e^{\frac{-r}{2 a_{0}}}$ where A is a constant and $a_{0}$ is the Bohr radius. Using this, calculate the most probable value of $r$ (the distance from the nucleus to the electron) in this state. ans: $4 a_{0}$
C. In class, I found the two lowest energy levels of an electron confined to a small cube. For a cube 2.00 $\AA \AA$ on a side, (a) Find the next three energies above those. (b) How many states are there with each energy you found in (a)? Give three answers - one for each energy.
ans: $84.6 \mathrm{eV}, 3$ states; $103 \mathrm{eV}, 3$ states; $113 \mathrm{eV}, 1$ state
D. 1. (2 pts) According to Bohr's theory, what is the orbital angular momentum of a hydrogen atom in a 1s state? According to the Schrodinger treatment, what is the orbital angular momentum of a hydrogen
atom in a 1s state?
2. (8) a. Write out all possible combinations of the quantum numbers $\ell$ and $\mathrm{m}_{\ell}$ for $\mathrm{He}^{+}$ion $(\mathrm{z}=2)$ with n $=3$. Express each combination as a pair of numbers: $\ell, \mathrm{m}_{\ell}(\mathrm{b})$ What is the energy of these states? ans: $\ell, \mathrm{m}_{\ell}=0,0 ; 1,-1 ; 1,0 ; 1,1 ; 2,-2 ; 2,-1 ; 2,0 ; 2,1 ;$ or 2,2 . Energy of all $\mathrm{n}=3$ states is -6.04 eV .
E. The ground state wave function for hydrogen is $\psi_{1 s}=\frac{1}{\sqrt{\pi a_{0}^{3}}} e^{-\frac{r}{a_{0}}}$, where $a_{0}$ is the Bohr radius. Show that this is normalized.

Hints:

- You can't have a negative radius. Integrate from 0 to $\infty$, not $-\infty$ to $\infty$.
- At the upper limit of integration $(r=\infty)$, you will have a polynomial times an exponential. An exponential dominates over any polynomial at infinity. (Use the value of the exponential and ignore the polynomial.)
F. For convenience, work in a system of units where $a_{0}=1$. This makes hydrogen's ground state wave function $\psi_{1 s}=\frac{1}{\sqrt{\pi}} e^{-r}$. What is the probability of finding the electron between $\mathrm{r}=0$ and $\mathrm{r}=\frac{1}{5}$ ? ans: . 00793


## Sec. 10 - Electromagnetic Waves

Read: Ch. 34: sec. 2-6
A. The average intensity of solar radiation at the Earth (above the atmosphere) is $1340 \mathrm{~W} / \mathrm{m}^{2}$. The average Earth-Sun distance is $1.496 \times 10^{11} \mathrm{~m}$. Calculate
(a) the total power radiated by the sun.
(b) the maximum values of the radiation's electric and magnetic fields at Earth. ans: $3.77 \times 10^{26} \mathrm{~W}, 1.01 \mathrm{kV} / \mathrm{m}, 3.35 \mu \mathrm{~T}$
B. 1. (1 point) . Multiple choice: In classical physics, any time a charge accelerates, it gives off $\qquad$ .
a. electromagnetic radiation
b. a constantly increasing magnetic field
c. electrons
d. a "displacement current"
2. (9) The electromagnetic wave is propagating in the $x$ direction. The wavelength is 50.0 m , and the electric field vibrates in the xy plane with an amplitude of 22.0 V/m.
(a) Find its frequency.
(b) Find the magnitude and direction of $\vec{B}$ when the electric field has its maximum value in the negative $y$ direction.

(c) Write a expression for B in the form $\mathrm{B}=\mathrm{B}_{\max } \cos (\mathrm{kx}-\omega t)$.
ans: $6.00 \mathrm{MHz},-73.3 \hat{\mathrm{k}} \mathrm{nT}, \mathrm{B}=(73.3 \mathrm{nT}) \cos \left(.126 \mathrm{x}-3.77 \times 10^{7} \mathrm{t}\right)$
C. 1. (2 pts) Describe the physical significance of the Poynting vector.
2. (8) A 4000 kg spaceship is to be propelled by using a "sail" which reflects solar radiation. The sail is totally reflecting, oriented with its plane perpendicular to the direction of the sun, and $1.00 \mathrm{~km} \times 1.50 \mathrm{~km}$ in size. What is the maximum acceleration that the radiation can give this ship, if its intensity is 1340 $\mathrm{W} / \mathrm{m}^{2}$ ?
ans: $3.35 \mathrm{~mm} / \mathrm{s}^{2}$
D. 1. ( 2 pts ) If you charge a comb by running it through your hair, then hold the comb next to a bar magnet, do these electric and magnetic fields constitute an electromagnetic wave?
2. (8) At a particular distance from the sun, the rms value of the magnetic field caused by its radiation is $1.80 \mu \mathrm{~T}$. For this solar radiation, calculate
(a) the magnitude of the rms electric field.
(b) the average energy density.
(c) the average magnitude of the Poynting vector.
ans: $540 \mathrm{~V} / \mathrm{m}, 2.58 \mu \mathrm{~J} / \mathrm{m}^{3}, 774 \mathrm{~W} / \mathrm{m}^{2}$
E. A 15.0 mW helium-neon laser $(\lambda=632.8 \mathrm{~nm})$ emits a beam of circular cross-section with a diameter of 2.00 mm .
(a) Find the maximum electric field in the beam.
(b) What total energy is contained in a 1.00 m length of the beam?
ans: $1.90 \mathrm{kV} / \mathrm{m}, 50.0 \mathrm{pJ}$
F. From the equations describing an electromagnetic wave, $E=E_{m} \sin [(\omega / c) x-\omega t]$ and $B=B_{m} \sin [(\omega / c) x-$ $\omega t$, and also from the relevant Maxwell equations (in differential form),

$$
\frac{\partial \mathrm{E}}{\partial \mathrm{x}}=-\frac{\partial \mathrm{B}}{\partial \mathrm{t}} \text { and } \frac{\partial \mathrm{B}}{\partial \mathrm{x}}=-\mu_{0} \varepsilon_{0} \frac{\partial \mathrm{E}}{\partial \mathrm{t}}
$$

show that the speed of light in a vacuum is $\mathrm{c}=1 / \sqrt{\mu_{o} \varepsilon_{o}}$. (Work from just the relationships mentioned here. The formulas on the formula sheet are what you're supposed to be proving.)

## Sec. 11 - Multi-Electron Atoms/Molecules/Solids

Read: Skim Ch. $21 \sec 3 ;$ Ch. $42 \sec 7$; Ch. $43 \sec 2-6$ (Just skim sec 3 )
A. 1. (2 pts) An unexcited Boron atom contains two 1s electrons, two 2 s electrons and one more. Give
one of the possibilities for the quantum numbers of this last electron. $\mathrm{n}=$ $\qquad$ , $\ell=$ $\qquad$ , $\mathrm{m}_{\ell}=$ $\qquad$ $\& \mathrm{~m}_{\mathrm{s}}=$ $\qquad$ .
2. At absolute zero, the number of electrons in a metal per unit volume with energies between E and $E+d E$ reduces to

$$
N(E) d E= \begin{cases}\frac{\pi(8 m)^{3 / 2}}{2 h^{3}} \sqrt{E} d E & \text { for } 0<E<E_{F} \\ 0 & \text { for } E_{F}<E<\infty\end{cases}
$$

Use this and the fact that $n=\int_{0}^{\infty} N(E) d E$ to show that $E_{F}=\frac{h^{2}}{8 m}\left(\frac{3 n}{\pi}\right)^{2 / 3}$
B. 1. (2 pts) Why does a diatomic gas have a greater energy content per mole than a monatomic gas at the same temperature?
2. (8) Molecule Photon Frequency (Hz) Force Constant, k (N/m)

| HI | $6.69 \times 10^{13}$ | 320 |
| :--- | :--- | :--- |
| HF | $8.72 \times 10^{13}$ | 970 |

Use data from this table to calculate the minimum amplitude of vibration for (a) the HI molecule, and (b) the HF molecule. (c) Which has the weaker bond?
ans: $.0118 \mathrm{~nm}, .00772 \mathrm{~nm}, \mathrm{HI}$ is less stiff
C. 1. (2 pts) In a metal at room temperature,
a. About what percentage of the states that are .5 eV below the Fermi Energy are occupied?
b. About what percentage of the states that are .5 eV above the Fermi Energy are occupied?
2. (8) a. Write out the electronic configuration for the ground state of sodium $(Z=11)$.
b. Write out the values for the set of quantum numbers $\mathrm{n}, \ell, \mathrm{m}_{\ell}$ and $\mathrm{m}_{\mathrm{s}}$ for each electron in sodium.
ans: a. $1 s^{2} 2 s^{2} 2 p^{6} 3 s^{1}$ b. See solutions.
D. Sodium is a monovalent metal having a density of $.971 \mathrm{~g} / \mathrm{cm}^{3}$ and molar mass of $23.0 \mathrm{~g} / \mathrm{mole}$. Use this to calculate
a. the density of charge carriers, and
b. the Fermi energy.
ans: $2.54 \times 10^{28}$ electrons $/ \mathrm{m}^{3}, 3.15 \mathrm{eV}$
E. A HCl molecule is excited to its first rotational energy level, corresponding to $\mathrm{J}=1$. If the distance between its nuclei is .1275 nm , what is the angular speed of the molecule about its center of mass? Assume the isotopes involved are ${ }^{1} \mathrm{H}$ and ${ }^{35} \mathrm{Cl}$.
ans: 5.69 Trad/s
F. Calculate the energy of a conduction electron in silver at 800 K if the probability of finding an electron in that state is .950 . The Fermi energy is 5.48 eV at this temperature.
ans: 5.28 eV

## Sec. 12 - More on Solids/The Nucleus

Read: Ch. 27 sec 5 ; p. 880 ; Ch. $43 \sec 4-8$; Ch. $44 \sec 1 \& 2$. (The book does not include all I will cover about superconductors, so be sure you have good notes.)
A. 1. ( $11 / 2 \mathrm{pts}$ ) Put "insulator," "semiconductor" or "metal" in the blank.

- In a(n) $\qquad$ , the highest energy band containing electrons (the conduction band) is partly filled with many electrons.
- A(n) $\qquad$ has an empty conduction band above a filled valence band.
- A(n) $\qquad$ has an empty conduction band above a filled valence band at absolute zero,
but the gap between bands is narrow enough for some electrons to cross it at room temperature.

2. (2 pts) Why do lattice imperfections and lattice vibrations (phonons), which scatter electrons in normal metals, have no effect on Cooper pairs?
3. ( $3^{1 ⁄ 2}$ ) A small permanent magnet is placed above a superconductor. (a) What happens to the superconductor to keep $\mathrm{B}=0$ inside of it? (b) How does this explain the levitation of the magnet?
4. (3) Find the radius of a nucleus of ${ }_{92}^{238} \mathrm{U}$.
ans: 7.44 fm
B. 1. (1.5 points) What happens to a superconductor when it is placed in a magnetic field which is stronger than $B_{c}$, its critical magnetic field?
5. (1.5) A small cube of superconducting material would have energy levels like those of a three dimensional square well. How many Cooper Pairs will be in the lowest of these energy levels, $\mathrm{E}_{111}$ ?
6. (7) Using the fact that the atomic mass of ${ }^{56}{ }_{26} \mathrm{Fe}$ is 55.934940 , find its binding energy per nucleon.
ans: $8.79 \mathrm{MeV} /$ nucleon
C. 1. (3 pts) Explain what goes on in a p-n junction that allows current to flow through it in one direction, but practically stops it from flowing the other way.
7. (7) Light emitted by hydrogen falls on a CdS crystal (energy gap $=2.42 \mathrm{eV}$ ). Which lines from the Balmer series are transmitted through the CdS and which does it absorb? (A substance's band structure determines optical properties such as its color. CdS is orange because photons at that end of the spectrum have too little energy to get an electron across the 2.42 eV gap and are not absorbed. High frequencies do have enough energy to excite electrons. Remove blue from white light and orange is left.)
ans: The $\mathrm{n}=3$ to $\mathrm{n}=2$ line is transmitted, all others absorbed.
D. Consider a cube 1.30 mm on an edge, made of gold (Fermi Energy $=5.53 \mathrm{eV}$ ). Calculate the approximate number of conduction electrons in this cube whose energies lie in the range 5.4900 to 5.4904 eV , at 300 K .
ans: $1.15 \times 10^{16}$
E. 1. (2 points) The visible part of the spectrum includes photons from about 1.8 eV to about 3.1 eV . At room temperature, silicon has an energy gap of 1.14 eV and diamond has an energy gap of 5.47 eV . Explain why silicon is opaque but diamond is transparent. (What happens when a photon tries to interact with an electron in each material?)
8. (8) Consider a piece of gold (Fermi Energy $=5.53 \mathrm{eV}$ ). Calculate the approximate number of conduction electrons per cubic meter whose energies lie in the range 2.00 to 4.00 eV , at 300 K .
ans: $2.35 \times 10^{28}$
F. 1. (2 points) Silicon atoms each atom have four electrons to use bonding with neighboring atoms.
a. Explain why doping Si with phosphorous, which has 5 valence electrons, makes it n-silicon.
b. Explain why doping Si with boron, which has 3 valence electrons, makes it p -silicon.
9. ( 2 pts ) A common type of transistor is a pnp or npn sandwich, whose center layer is very thin. Explain how this acts as a "valve" for electric current.
10. (1) Is the center layer of a pnp transistor doped with donors or acceptors?
11. (2) How is it possible for all Cooper pairs in a superconductor to be in the same quantum state?
12. (1.5) What happens to the Cooper pairs in a superconductor if it is heated above the critical temperature?
13. (1.5) What sort of experiment first indicated that the atom has a small positive nucleus, containing most of the mass?

## Sec. 13 - Radioactivity \& Nuclear Reactions

Read: Ch. 44, sec 4, 5 \& 7; Ch. 45: sec. 2, skim sec 3, sec. 4 to bottom p. 1136, sec. 5.
A. 1. (2 pts) Why are heavy nuclei unstable?
2. (8) Using this graph, estimate the energy released when a nucleus of mass number 200 is split into two nuclei each of mass number 100.
ans: about 180 MeV , depending on how you estimate tenths of an MeV on the graph.

B. A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi . After 4.00 h , its activity is 8.00 mCi .
a. Find the decay constant and half life.
b. How many atoms of the isotope were contained in the freshly prepared sample?
ans: $1.55 \times 10^{-5} \mathrm{~s}^{-1}, 12.4 \mathrm{~h}, 2.39 \times 10^{13}$
C. 1. (4 pts) Explain the difference between alpha, beta and gamma rays. (I don't expect you to list every single way they differ, so various answers will be considered correct.)
2. (6) Determine which decays can occur spontaneously. Show why each answer is correct.
(a) ${ }^{40}{ }_{20} \mathrm{Ca} \rightarrow \mathrm{e}^{+}+{ }^{40}{ }_{19} \mathrm{~K}$
(b) ${ }^{99}{ }_{44} \mathrm{Ru} \rightarrow{ }_{2}^{4} \mathrm{He}+{ }_{42}^{95} \mathrm{Mo}$
(c) ${ }_{60}^{14} \mathrm{Nd} \rightarrow{ }_{2}^{4} \mathrm{He}+{ }^{140}{ }_{58} \mathrm{Ce}$
ans: Only (c) can occur.

| Name | Symbol | Mass <br> Number | Atomic Mass <br> $(\mathrm{u})$ |
| :---: | :---: | :---: | :---: |
| Electron | e | 0 | .000549 |
| Helium | He | 3 | 3.016029 |
|  | He | 4 | 4.002603 |
| Potassium | K | 39 | 38.963707 |
|  | K | 40 | 39.963999 |
| Calcium | Ca | 40 | 39.962591 |
| Molybdenum | Mo | 95 | 94.905842 |
| Ruthenium | Ru | 99 | 98.905939 |
| Cerium | Ce | 140 | 139.905434 |
| Neodymium | Nd | 142 | 141.907719 |
|  | Nd | 144 | 143.910083 |

D. Consider a sample of ${ }^{239}{ }_{94} \mathrm{Pu}$, which has a half life of 24120 years, and a mass of 1.00 kg at $\mathrm{t}=0$. Calculate
(a) the number of ${ }^{239}{ }_{94} \mathrm{Pu}$ nuclei present at $\mathrm{t}=0$,
(b) the initial activity in the sample,
(c) how much time until it reaches a "safe" activity of 0.100 Bq . ans: $2.52 \times 10^{24}, 2.29 \mathrm{TBq}, 1.07 \mathrm{Myr}$
E. 1. ( 2 pts ) If a radioactive material has a half-life of one year, does this mean it will be completely decayed after two years? Explain.
2. (2) What is given off by the fission of uranium that can go on to cause another fission event?
2. (6) Identify the missing nuclide or particle, X . (If this is the quiz, a periodic table of the elements will be printed on the back.)
(a) $\mathrm{X} \rightarrow{ }^{65}{ }_{28} \mathrm{Ni}+\gamma$
(b) ${ }^{215}{ }_{84} \mathrm{Po} \rightarrow \mathrm{X}+\alpha$
(c) $\mathrm{X} \rightarrow{ }_{26}^{55} \mathrm{Fe}+\mathrm{e}^{+}+v$

$$
\text { ans: }{ }_{28}^{65 \mathrm{Ni},}{ }^{211}{ }_{82} \mathrm{~Pb},{ }_{27}^{55} \mathrm{Co}
$$

F. 1. (4) In terms of biological damage, how many rad of heavy ions is equivalent to 10 rad of x -rays? ans: . 5 rad
2. (6) During a break, a technician decides to heat 100 g of water for some coffee with his x-ray machine. If the machine produces $10.0 \mathrm{rad} / \mathrm{s}$, what will the water's temperature increase be after five minutes? (The
specific heat of water is $4186 \mathrm{~J} / \mathrm{kg} .{ }^{\circ} \mathrm{C}$.)
ans: $.00717^{\circ} \mathrm{C}$

## Sec. 14 - Elementary Particles/Cosmology

Read: Ch. 39 sec. 9; Ch. 46, all (I'll be skimming in places.)
A. 1. ( 2 pts ) What is an important difference between baryons and mesons?
2. (8) For each reaction, state the quark composition of each particle, and the total number of up quarks, down quarks, and strange quarks, both before and after. (For example, count an up and an anti-up as a total of zero.) In part (d), identify the mystery particle.
(a) $\pi^{-}+\mathrm{p} \rightarrow \mathrm{K}^{0}+\Lambda^{0}$
(b) $\pi^{+}+\mathrm{p} \rightarrow \mathrm{K}^{+}+\Sigma^{+}$
(c) $\mathrm{K}^{-}+\mathrm{p} \rightarrow \mathrm{K}^{+}+\mathrm{K}^{0}+\Omega^{-}$
(d) $\mathrm{p}+\mathrm{p} \rightarrow \mathrm{K}^{0}+\mathrm{p}+\pi^{+}+$?
ans: See solutions; $\Lambda^{0}$
B. 1. (1 point) The muon was once known as the "mu meson." Why did the name need to be changed?
2. (2 pts) What is the source of the cosmic microwave background?
3. (3) A distant quasar is moving rapidly away from the Earth. The shift in its spectral lines corresponds to a speed of .160 c . Determine the distance from Earth to this quasar.
ans: $2.18 \times 10^{9} \mathrm{ly}$
4. (4) Occasionally, high energy muons will collide with electrons and produce two neutrinos according to the reaction $\mu^{+}+\mathrm{e} \rightarrow 2 v$. What kind of neutrinos are these?

$$
\text { ans: } v_{e}, \bar{v}_{\mu}
$$

C. Determine which of the following reactions can occur. For those that can't, name a conversation law which is violated.
(a) $p \rightarrow \pi^{+}+\pi^{0}$
(e) $\pi^{+} \rightarrow \mu^{+}+{ }^{v} \mu$
(b) $\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\mathrm{p}+\pi^{0}$
(f) $\mathrm{n} \rightarrow \mathrm{p}+\mathrm{e}^{-}+\bar{\nu}_{e}$
(c) $\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\mathrm{p}+\pi^{+}$
(d) $\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\pi^{+}$
(g) $\pi^{+} \rightarrow \mu^{+}+n$
2. (4) Name the four fundamental interactions, and the field particle that mediates each.
3. (5) Give the color and flavor of all the quarks in
a. A $\mathrm{K}^{\circ}$ meson containing an antiblue $\overline{\mathrm{s}}$ quark.
b. A proton containing a blue d quark and green u quark.
c. A muon.
ans. blue $d \&$ antiblue $\bar{s}$; red $u$, green $u$, blue $d$; none
E. 1. In the absence of gravity, the beam of light follows a horizontal line when the elevator is stationary. At $t=0$, the elevator acquires an upward acceleration of $g$. In terms of $x, g$ and $c$, what is $y$, the deflection of the spot on the wall? (It's simplest to consider a photon emitted at $\mathrm{t}=0$ and calculate the distance the wall moves up while the photon is traveling.) ans: $y=-1 / 2 g(x / c)^{2}$

2. In an elevator at rest, what distance would a gravitational field of strength $g$ bend the beam downward? ans: $y=-1 / 2 g(x / c)^{2}$
3. A crude model of the sun's gravitational field is shown. (No gravity outside the dashed circle, a uniform field inside.) Find the angle $\theta$ by which the ray of starlight is deflected. (Give a numerical value, not a formula.) Hint: The ray's path through the field is given by the function you found in part 2. As shown at lower right, $\tan \theta=d y / d x$ as the ray leaves the field.
ans: $6.67 \times 10^{-6} \mathrm{rad}$, or $1.38^{\prime \prime}$ (similar to the $1.75 "$ observed in 1919.)

(There is no quiz F this week.)

