# Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**Finding the Derivative by the Limit Process** **In Exercises 1–4, find the derivative of the function by the limit process.**

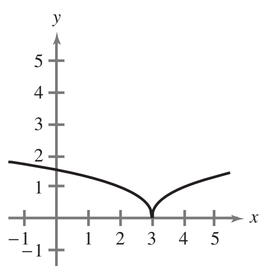
1. $f(x) = 12$
2. $f(x) = 5x - 4$
3. $f(x) = x^2 - 4x + 5$
4. $f(x) = \dfrac{6}{x}$

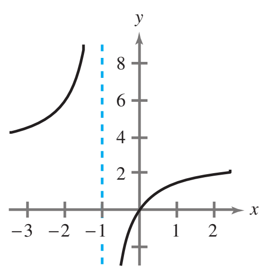
**Using the Alternative Form of the Derivative** **In Exercises 5 and 6, use the alternative form of the derivative to find the derivative at $x = c$ (if it exists).**

5. $g(x) = 2x^2 - 3x, \quad c = 2$
6. $f(x) = \dfrac{1}{x+4}, \quad c = 3$

**Determining Differentiability** **In Exercises 7 and 8, describe the $x$-values at which $f$ is differentiable.**

7. $f(x) = (x - 3)^{2/5}$
8. $f(x) = \dfrac{3x}{x+1}$




**Finding a Derivative** **In Exercises 9–20, use the rules of differentiation to find the derivative of the function.**

9. $y = 25$
10. $f(t) = 4t^4$
11. $f(x) = x^3 - 11x^2$
12. $g(s) = 3s^5 - 2s^4$
13. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$
14. $f(x) = x^{1/2} - x^{-1/2}$
15. $g(t) = \dfrac{2}{3t^2}$
16. $h(x) = \dfrac{8}{5x^4}$
17. $f(\theta) = 4\theta - 5\sin\theta$
18. $g(\alpha) = 4\cos\alpha + 6$
19. $f(\theta) = 3\cos\theta - \dfrac{\sin\theta}{4}$
20. $g(\alpha) = \dfrac{5\sin\alpha}{3} - 2\alpha$

**Finding the Slope of a Graph** **In Exercises 21–24, find the slope of the graph of the functions at the given point.**

21. $f(x) = \dfrac{27}{x^3}, \quad (3, 1)$
22. $f(x) = 3x^2 - 4x, \quad (1, -1)$
23. $f(x) = 2x^4 - 8, \quad (0, -8)$
24. $f(\theta) = 3\cos\theta - 2\theta, \quad (0, 3)$

25. **Vibrating String** When a guitar string is plucked, it vibrates with a frequency of $F = 200\sqrt{T}$, where $F$ is measured in vibrations per second and the tension $T$ is measured in pounds. Find the rates of change of $F$ when (a) $T = 4$ and (b) $T = 9$.

26. **Volume** The surface area of a cube with sides of length $\ell$ is given by $S = 6\ell^2$. Find the rates of change of the surface area with respect to $\ell$ when (a) $\ell = 3$ inches and (b) $\ell = 5$ inches.

**Vertical Motion** **In Exercises 27 and 28, use the position function $s(t) = -16t^2 + v_0t + s_0$ for free-falling objects.**

$S_0 = 600$

$V_0 = -30$

27. A ball is thrown straight down from the top of a 600-foot building with an initial velocity of $-30$ feet per second.
   (a) Determine the position and velocity functions for the ball.
   ~~(b)~~ Determine the average velocity on the interval $[1, 3]$.
   (c) Find the instantaneous velocities when $t = 1$ and $t = 3$.
   (d) Find the time required for the ball to reach ground level.
   (e) Find the velocity of the ball at impact.

28. To estimate the height of a building, a weight is dropped from the top of the building into a pool at ground level. The splash is seen 9.2 seconds after the weight is dropped. What is the height (in feet) of the building?

**Finding a Derivative** **In Exercises 29–40, use the Product Rule or the Quotient Rule to find the derivative of the function.**

29. $f(x) = (5x^2 + 8)(x^2 - 4x - 6)$
30. $g(x) = (2x^3 + 5x)(3x - 4)$
31. $h(x) = \sqrt{x}\sin x$
32. $f(t) = 2t^5\cos t$
33. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$
34. $f(x) = \dfrac{2x + 7}{x^2 + 4}$
35. $y = \dfrac{x^4}{\cos x}$
36. $y = \dfrac{\sin x}{x^4}$
37. $y = 3x^2\sec x$
38. $y = 2x - x^2\tan x$
39. $y = x\cos x - \sin x$
40. $g(x) = 3x\sin x + x^2\cos x$

**Finding an Equation of a Tangent Line** **In Exercises 41–44, find an equation of the tangent line to the graph of $f$ at the given point.**

41. $f(x) = (x + 2)(x^2 + 5), \quad (-1, 6)$
42. $f(x) = (x - 4)(x^2 + 6x - 1), \quad (0, 4)$
43. $f(x) = \dfrac{x+1}{x-1}, \quad \left(\dfrac{1}{2}, -3\right)$
44. $f(x) = \dfrac{1 + \cos x}{1 - \cos x}, \quad \left(\dfrac{\pi}{2}, 1\right)$

**Finding a Second Derivative** **In Exercises 45–50, find the second derivative of the function.**

45. $g(t) = -8t^3 - 5t + 12$
46. $h(x) = 6x^{-2} + 7x^2$
47. $f(x) = 15x^{5/2}$
48. $f(x) = 20\sqrt[5]{x}$
49. $f(\theta) = 3\tan\theta$
50. $h(t) = 10\cos t - 15\sin t$

**51. Acceleration** The velocity of an object in meters per second is $v(t) = 20 - t^2$, $0 \le t \le 6$. Find the velocity and acceleration of the object when $t = 3$.

**52. Acceleration** The velocity of an automobile starting from rest is

$$v(t) = \frac{90t}{4t + 10}$$

where $v$ is measured in feet per second. Find the acceleration at (a) 1 second, (b) 5 seconds, and (c) 10 seconds.

**Finding a Derivative** **In Exercises 53–64, find the derivative of the function.**

**53.** $y = (7x + 3)^4$

**54.** $y = (x^2 - 6)^3$

**55.** $y = \dfrac{1}{x^2 + 4}$

**56.** $f(x) = \dfrac{1}{(5x + 1)^2}$

**57.** $y = 5\cos(9x + 1)$

**58.** $y = 1 - \cos 2x + 2\cos^2 x$

**59.** $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$

**60.** $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

**61.** $y = x(6x + 1)^5$

**62.** $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

**63.** $f(x) = \dfrac{3x}{\sqrt{x^2 + 1}}$

**64.** $h(x) = \left(\dfrac{x + 5}{x^2 + 3}\right)^2$

**Evaluating a Derivative** **In Exercises 65–70, find and evaluate the derivative of the function at the given point.**

**65.** $f(x) = \sqrt{1 - x^3}$, $(-2, 3)$

**66.** $f(x) = \sqrt[3]{x^2 - 1}$, $(3, 2)$

**67.** $f(x) = \dfrac{4}{x^2 + 1}$, $(-1, 2)$

**68.** $f(x) = \dfrac{3x + 1}{4x - 3}$, $(4, 1)$

**69.** $y = \dfrac{1}{2}\csc 2x$, $\left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$

**70.** $y = \csc 3x + \cot 3x$, $\left(\dfrac{\pi}{6}, 1\right)$

**Finding a Second Derivative** **In Exercises 71–74, find the second derivative of the function.**

**71.** $y = (8x + 5)^3$

**72.** $y = \dfrac{1}{5x + 1}$

**73.** $f(x) = \cot x$

**74.** $y = \sin^2 x$

**75. Refrigeration** The temperature $T$ (in degrees Fahrenheit) of food in a freezer is

$$T = \frac{700}{t^2 + 4t + 10}$$

where $t$ is the time in hours. Find the rate of change of $T$ with respect to $t$ at each of the following times.

(a) $t = 1$ (b) $t = 3$ (c) $t = 5$ (d) $t = 10$

**76. Harmonic Motion** The displacement from equilibrium of an object in harmonic motion on the end of a spring is

$$y = \frac{1}{4}\cos 8t - \frac{1}{4}\sin 8t$$

where $y$ is measured in feet and $t$ is the time in seconds. Determine the position and velocity of the object when $t = \pi/4$.

**Finding a Derivative** **In Exercises 77–82, find *dy/dx* by implicit differentiation.**

**77.** $x^2 + y^2 = 64$

**78.** $x^2 + 4xy - y^3 = 6$

**79.** $x^3y - xy^3 = 4$

**80.** $\sqrt{xy} = x - 4y$

**81.** $x \sin y = y \cos x$

**82.** $\cos(x + y) = x$

**Tangent Lines and Normal Lines** **In Exercises 83 and 84, find equations for the tangent line and the normal line to the graph of the equation at the given point. (The *normal line* at a point is perpendicular to the tangent line at the point.) Use a graphing utility to graph the equation, the tangent line, and the normal line.**

**83.** $x^2 + y^2 = 10$, $(3, 1)$

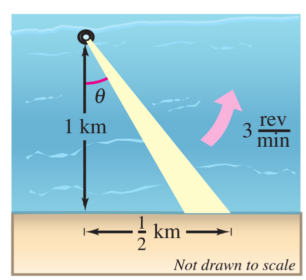
**84.** $x^2 - y^2 = 20$, $(6, 4)$

**85. Rate of Change** A point moves along the curve $y = \sqrt{x}$ in such a way that the $y$-value is increasing at a rate of 2 units per second. At what rate is $x$ changing for each of the following values?

(a) $x = \frac{1}{2}$ (b) $x = 1$ (c) $x = 4$

**86. Surface Area** All edges of a cube are expanding at a rate of 8 centimeters per second. How fast is the surface area changing when each edge is 6.5 centimeters?

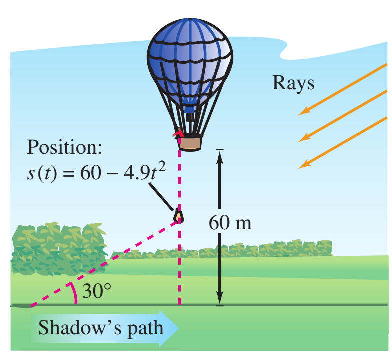
**87. Linear vs. Angular Speed** A rotating beacon is located 1 kilometer off a straight shoreline (see figure). The beacon rotates at a rate of 3 revolutions per minute. How fast (in kilometers per hour) does the beam of light appear to be moving to a viewer who is $\frac{1}{2}$ kilometer down the shoreline?




**88. Moving Shadow** A sandbag is dropped from a balloon at a height of 60 meters when the angle of elevation to the sun is 30° (see figure). The position of the sandbag is

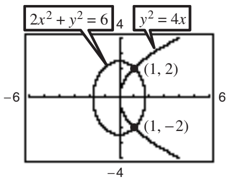
$$s(t) = 60 - 4.9t^2.$$

Find the rate at which the shadow of the sandbag is traveling along the ground when the sandbag is at a height of 35 meters.

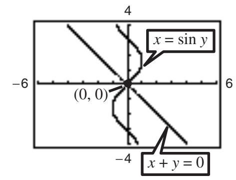


**55.** $x^2 + y^2 = r^2 \Rightarrow y' = -x/y \Rightarrow y/x =$ slope of normal line. Then for $(x_0, y_0)$ on the circle, $x_0 \neq 0$, an equation of the normal line is $y = (y_0/x_0)x$, which passes through the origin. If $x_0 = 0$, the normal line is vertical and passes through the origin.

**57.** Horizontal tangents: $(-4, 0)$, $(-4, 10)$
Vertical tangents: $(0, 5)$, $(-8, 5)$

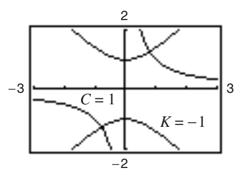
**59.**


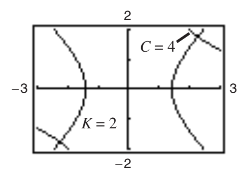

At $(1, 2)$:
Slope of ellipse: $-1$
Slope of parabola: $1$
At $(1, -2)$:
Slope of ellipse: $1$
Slope of parabola: $-1$

**61.**


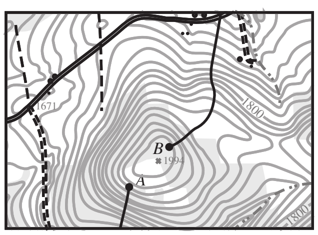

At $(0, 0)$:
Slope of line: $-1$
Slope of sine curve: $1$

**63.** Derivatives: $\dfrac{dy}{dx} = -\dfrac{y}{x}, \dfrac{dy}{dx} = \dfrac{x}{y}$

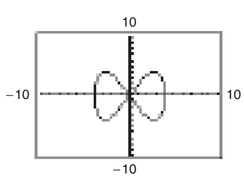


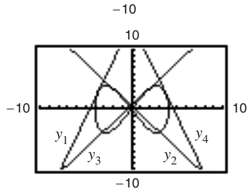




**65.** Answers will vary. In the explicit form of a function, the variable is explicitly written as a function of $x$. In an implicit equation, the function is only implied by an equation. An example of an implicit function is $x^2 + xy = 5$. In explicit form, it would be $y = (5 - x^2)/x$.

**67.**



Use starting point $B$.

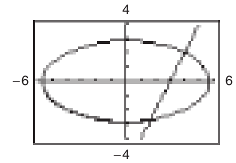
**69.** (a)



(b)



$y_1 = \frac{1}{3}\left[(\sqrt{7} + 7)x + (8\sqrt{7} + 23)\right]$
$y_2 = -\frac{1}{3}\left[(-\sqrt{7} + 7)x - (23 - 8\sqrt{7})\right]$
$y_3 = -\frac{1}{3}\left[(\sqrt{7} - 7)x - (23 - 8\sqrt{7})\right]$
$y_4 = -\frac{1}{3}\left[(\sqrt{7} + 7)x - (8\sqrt{7} + 23)\right]$

(c) $\left(\dfrac{8\sqrt{7}}{7}, 5\right)$

**71.** Proof **73.** $y = -\dfrac{\sqrt{3}}{2}x + 2\sqrt{3}$, $y = \dfrac{\sqrt{3}}{2}x - 2\sqrt{3}$

**75.** (a) $y = 2x - 6$
(b)



(c) $\left(\frac{28}{17}, -\frac{46}{17}\right)$

## Section 2.6 *(page 153)*

**1.** (a) $\frac{3}{4}$ (b) 20 **3.** (a) $-\frac{5}{8}$ (b) $\frac{3}{2}$
**5.** (a) $-8$ cm/sec (b) 0 cm/sec (c) 8 cm/sec
**7.** (a) 12 ft/sec (b) 6 ft/sec (c) 3 ft/sec
**9.** In a linear function, if $x$ changes at a constant rate, so does $y$. However, unless $a = 1$, $y$ does not change at the same rate as $x$.
**11.** (a) $64\pi$ cm$^2$/min (b) $256\pi$ cm$^2$/min
**13.** (a) $972\pi$ in.$^3$/min; $15{,}552\pi$ in.$^3$/min
(b) If $dr/dt$ is constant, $dV/dt$ is proportional to $r^2$.
**15.** (a) 72 cm$^3$/sec (b) 1800 cm$^3$/sec
**17.** $8/(405\pi)$ ft/min **19.** (a) 12.5% (b) $\frac{1}{144}$ m/min
**21.** (a) $-\frac{7}{12}$ ft/sec; $-\frac{3}{2}$ ft/sec; $-\frac{48}{7}$ ft/sec
(b) $\frac{527}{24}$ ft$^2$/sec (c) $\frac{1}{12}$ rad/sec
**23.** Rate of vertical change: $\frac{1}{5}$ m/sec
Rate of horizontal change: $-\sqrt{3}/15$ m/sec
**25.** (a) $-750$ mi/h (b) 30 min
**27.** $-50/\sqrt{85} \approx -5.42$ ft/sec
**29.** (a) $\frac{25}{3}$ ft/sec (b) $\frac{10}{3}$ ft/sec
**31.** (a) 12 sec (b) $\frac{1}{2}\sqrt{3}$ m (c) $\sqrt{5}\pi/120$ m/sec
**33.** Evaporation rate proportional to $S \Rightarrow \dfrac{dV}{dt} = k(4\pi r^2)$

$V = \left(\dfrac{4}{3}\right)\pi r^3 \Rightarrow \dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$. So $k = \dfrac{dr}{dt}$.

**35.** 0.6 ohm/sec **37.** $\dfrac{dv}{dt} = \dfrac{16r}{v}\sec^2\theta\dfrac{d\theta}{dt}, \dfrac{d\theta}{dt} = \dfrac{v}{16r}\cos^2\theta\dfrac{dv}{dt}$
**39.** $\dfrac{2\sqrt{21}}{525} \approx 0.017$ rad/sec
**41.** (a) $\dfrac{200\pi}{3}$ ft/sec (b) $200\pi$ ft/sec
(c) About $427.43\pi$ ft/sec
**43.** About 84.9797 mi/h
**45.** (a) $\dfrac{dy}{dt} = 3\dfrac{dx}{dt}$ means that $y$ changes three times as fast as $x$ changes.
(b) $y$ changes slowly when $x \approx 0$ or $x \approx L$. $y$ changes more rapidly when $x$ is near the middle of the interval.
**47.** $-18.432$ ft/sec$^2$ **49.** About $-97.96$ m/sec

## Review Exercises for Chapter 2 *(page 157)*

**1.** $f'(x) = 0$ **3.** $f'(x) = 2x - 4$ **5.** 5
**7.** $f$ is differentiable at all $x \neq 3$. **9.** 0 **11.** $3x^2 - 22x$
**13.** $\dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$ **15.** $-\dfrac{4}{3t^3}$ **17.** $4 - 5\cos\theta$
**19.** $-3\sin\theta - (\cos\theta)/4$ **21.** $-1$ **23.** 0
**25.** (a) 50 vibrations/sec/lb (b) 33.33 vibrations/sec/lb

**27.** (a) $s(t) = -16t^2 - 30t + 600$
$v(t) = -32t - 30$
(b) $-94$ ft/sec
(c) $v'(1) = -62$ ft/sec; $v'(3) = -126$ ft/sec
(d) About 5.258 sec (e) About $-198.256$ ft/sec

**29.** $4(5x^3 - 15x^2 - 11x - 8)$ **31.** $\sqrt{x}\cos x + \sin x/\left(2\sqrt{x}\right)$

**33.** $\dfrac{-(x^2+1)}{(x^2-1)^2}$ **35.** $\dfrac{4x^3\cos x + x^4 \sin x}{\cos^2 x}$

**37.** $3x^2 \sec x \tan x + 6x \sec x$ **39.** $-x \sin x$

**41.** $y = 4x + 10$ **43.** $y = -8x + 1$ **45.** $-48t$

**47.** $\frac{225}{4}\sqrt{x}$ **49.** $6 \sec^2 \theta \tan \theta$

**51.** $v(3) = 11$ m/sec; $a(3) = -6$ m/sec$^2$ **53.** $28(7x+3)^3$

**55.** $-\dfrac{2x}{(x^2+4)^2}$ **57.** $-45 \sin(9x+1)$

**59.** $\frac{1}{2}(1 - \cos 2x) = \sin^2 x$ **61.** $(36x+1)(6x+1)^4$

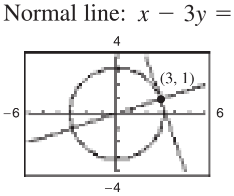
**63.** $\dfrac{3}{(x^2+1)^{3/2}}$ **65.** $\dfrac{-3x^2}{2\sqrt{1-x^3}}$; $-2$ **67.** $-\dfrac{8x}{(x^2+1)^2}$; 2

**69.** $-\csc 2x \cot 2x$; 0 **71.** $384(8x+5)$ **73.** $2\csc^2 x \cot x$

**75.** (a) $-18.667°/\text{h}$ (b) $-7.284°/\text{h}$
(c) $-3.240°/\text{h}$ (d) $-0.747°/\text{h}$

**77.** $-\dfrac{x}{y}$ **79.** $\dfrac{y(y^2 - 3x^2)}{x(x^2 - 3y^2)}$ **81.** $\dfrac{y \sin x + \sin y}{\cos x - x \cos y}$

**83.** Tangent line: $3x + y - 10 = 0$
Normal line: $x - 3y = 0$




**85.** (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

**87.** $450\pi$ km/h

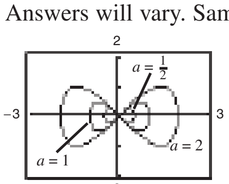
## P.S. Problem Solving *(page 159)*

**1.** (a) $r = \frac{1}{2}$; $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$
(b) Center: $\left(0, \frac{5}{4}\right)$; $x^2 + \left(y - \frac{5}{4}\right)^2 = 1$

**3.** $p(x) = 2x^3 + 4x^2 - 5$

**5.** (a) $y = 4x - 4$ (b) $y = -\frac{1}{4}x + \frac{9}{2}$; $\left(-\frac{9}{4}, \frac{81}{16}\right)$
(c) Tangent line: $y = 0$ (d) Proof
Normal line: $x = 0$

**7.** (a) Graph $\begin{cases} y_1 = \dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \\ y_2 = -\dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \end{cases}$ as separate equations.
(b) Answers will vary. Sample answer:




The intercepts will always be $(0, 0)$, $(a, 0)$, and $(-a, 0)$, and the maximum and minimum $y$-values appear to be $\pm\frac{1}{2}a$.
(c) $\left(\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$

**9.** (a) When the man is 90 ft from the light, the tip of his shadow is $112\frac{1}{2}$ ft from the light. The tip of the child's shadow is $111\frac{1}{9}$ ft from the light, so the man's shadow extends $1\frac{7}{18}$ ft beyond the child's shadow.
(b) When the man is 60 ft from the light, the tip of his shadow is 75 ft from the light. The tip of the child's shadow is $77\frac{7}{9}$ ft from the light, so the child's shadow extends $2\frac{7}{9}$ ft beyond the man's shadow.
(c) $d = 80$ ft
(d) Let $x$ be the distance of the man from the light, and let $s$ be the distance from the light to the tip of the shadow.
If $0 < x < 80$, then $ds/dt = -50/9$.
If $x > 80$, then $ds/dt = -25/4$.
There is a discontinuity at $x = 80$.

**11.** (a) $v(t) = -\frac{27}{5}t + 27$ ft/sec (b) 5 sec; 73.5 ft
$a(t) = -\frac{27}{5}$ ft/sec$^2$
(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

**13.** Proof. The graph of $L$ is a line passing through the origin $(0, 0)$.

**15.** (a) $j$ would be the rate of change of acceleration.
(b) $j = 0$. Acceleration is constant, so there is no change in acceleration.
(c) $a$: position function, $d$: velocity function,
$b$: acceleration function, $c$: jerk function

# Chapter 3

## Section 3.1 *(page 167)*

**1.** $f'(0) = 0$ **3.** $f'(2) = 0$ **5.** $f'(-2)$ is undefined.

**7.** 2, absolute maximum (and relative maximum)

**9.** 1, absolute maximum (and relative maximum);
2, absolute minimum (and relative minimum);
3, absolute maximum (and relative maximum)

**11.** $x = 0, x = 2$ **13.** $t = 8/3$ **15.** $x = \pi/3, \pi, 5\pi/3$

**17.** Minimum: $(2, 1)$
Maximum: $(-1, 4)$

**19.** Minimum: $(2, -8)$
Maximum: $(6, 24)$

**21.** Minimum: $\left(-1, -\frac{5}{2}\right)$
Maximum: $(2, 2)$

**23.** Minimum: $(0, 0)$
Maximum: $(-1, 5)$

**25.** Minimum: $(0, 0)$
Maxima: $\left(-1, \frac{1}{4}\right)$ and $\left(1, \frac{1}{4}\right)$

**27.** Minimum: $(1, -1)$
Maximum: $\left(0, -\frac{1}{2}\right)$

**29.** Minimum: $(-1, -1)$
Maximum: $(3, 3)$

**31.** Minimum value is $-2$ for $-2 \le x < -1$.
Maximum: $(2, 2)$

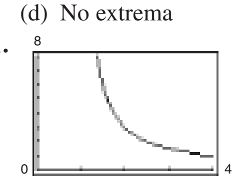
**33.** Minimum: $(3\pi/2, -1)$
Maximum: $(5\pi/6, 1/2)$

**35.** Minimum: $(\pi, -3)$
Maxima: $(0, 3)$ and $(2\pi, 3)$

**37.** (a) Minimum: $(0, -3)$;
Maximum: $(2, 1)$
(b) Minimum: $(0, -3)$
(c) Maximum: $(2, 1)$
(d) No extrema

**39.** (a) Minimum: $(1, -1)$;
Maximum: $(-1, 3)$
(b) Maximum: $(3, 3)$
(c) Minimum: $(1, -1)$
(d) Minimum: $(1, -1)$

**41.**




Minimum: $(4, 1)$