LAB 2 - Differentiation and Tangent Lines (20 points)

Due Date: Monday, March 8 (first two problems due Wed. Mar 3)

For each of the following equations, select a non-zero numerical value for \( k \) (where appropriate)

a) \( f(x) = \frac{2}{3} x^3 - 7x - k \)

b) \( f(x) = \frac{5-x}{kx+2} \)

c) \( f(x) = x^2 - 4x + 5\sin(x) \)

d) \( f(x) = \sec(x) + x \)

e) \( f(x) = \sqrt{x^3 - k} \)

1. Plot the equation using graph.cgi (see http://faculty.genesee.edu/kjmead for cgi programs)

2. Compute by hand, using differentiation techniques, the derivative \( \frac{dy}{dx} \) for this equation.
Remember that this derivative will tell you the slope of the tangent line at any point on the curve.

3. Find the coordinates of a point on the graph of \( f(x) \). Call this point \( (x_1,y_1) \) This point may be an approximation good to the tenths or hundredths place. (Show work).

4. Use your point \( (x_1,y_1) \) and your result for \( \frac{dy}{dx} \) to find the slope \( (m) \) of the tangent line to the curve at this point (again, good to one decimal place.)

5. Use your point \( (x_1,y_1) \) and your computed slope \( (m) \) to determine the equation of the tangent line to the curve at this point. Use the point slope form of the line: \( y - y_1 = m(x - x_1) \). Rewrite this equation in \( y=mx+b \) form.

6. Enter this equation in graph.cgi and plot. You should have plotted the tangent line to the graph of the original equation at \( (x_1,y_1) \). Zoom in on this point to make sure this line is close to the true tangent line. Since you may be approximating some numbers, your line may "just miss" the curve. This is OK, provided you didn't miss by much. If the graph of the line crosses the graph of \( f(x) \) at an angle of 20 degrees, then you have a problem and should figure out where you went wrong.

7. When you have plotted the tangent line, zoom in or out appropriately and print the screen containing your plot of the original equation, and the line. Label each line and tangent point.

On the same page as each graph, include the following information. Use the sample problem as a guideline for exactly how each problem should look:

a. The original equation

b. A neatly worked out solution for \( \frac{dy}{dx} \). Show any relevant work.

c. Your work for deriving equations for the tangent line. Show all details. Neatly label the graphs with their equations. Circle your answers.

Be Neat! Show all relevant calculations!

NOTE # 1: DO NOT CHOOSE POINTS WHERE THE TANGENT LINE HAS A SLOPE EQUAL TO ZERO!

NOTE # 2: Make sure you do your calculations for problem "c" and "d" IN RADIANS!!!

NOTE # 3: Make sure all work for your problem appears NEATLY on a single page.
**Function Grapher**

\[
\begin{align*}
 f(x) &= x^3 - 3x^2 - 4x - 1 \\
 g(x) &= 5(x-3) - 13 \\
 h(x) &= \quad \\
\end{align*}
\]

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**Zoom factor**
- x values
- y values
- in - out

**Original Window Size**

Center at: \((1, -8)\)

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\[
f(x) = x^3 - 3x^2 - 4x - 1
\]

I choose \(x = 3\) as my point, plug in for y-coordinate.

\[
f(3) = 3^3 - 3\cdot 3^2 - 4\cdot (3) - 1 = 27 - 27 - 12 - 1 = -13
\]

**Point of interest \((3, -13)\)**

Take derivative \(f'(x) = 3x^2 - 6x - 4\) \(\Rightarrow\) how work \(\text{for more complicated problems}\).

\[
f'(3) = 3\cdot 3^2 - 6\cdot 3 - 4 = 27 - 18 - 4 = 5
\]

\(m = 5\) (slope of tangent line)

Tangent line equation: \(y - y_1 = m(x - x_1)\) \(\Rightarrow\) plug into graph.cgi as \(5(x-3) + 13\)