

Calculus 1 - Lab 1 - Limits

Due: problems 1 & 2 due 9/14, rest by 9/21

Objectives:

1. To become familiar with a graphing utility such as "graph.cgi".
2. To develop an intuitive sense of the concept of a limit.
3. To use graphs to study one and two-sided limits.
4. To use algebra and calculus theorems to analyze one and two-sided limits.

Exercises: For each problem, you will examine a function $f(x)$ around a particular point $x = a$. You will use "limits.cgi" and "graph.cgi" from your instructor's website.

1. Enter your function $f(x)$ and the point $x=a$ into "limits.cgi". Print out this page.
2. Construct a simple graph of the function around $x=a$. Use graph.cgi. **Zoom in around the point $x=a$ in such a way that you can obtain a visual indication of the limit. Cut out the graph and paste it onto the upper right hand side of the page printed in part 1.**
3. Using the information gathered from above exercises, draw tentative conclusions about the values of the left-handed and right-handed limits. Write these conclusions and reasons on the printout from part one.
4. Use algebra, calculus, the numbers and the graph to answer all the questions on the printout from part one.
5. For the last question on the printout from part one, use the LIMIT THEOREMS and your knowledge of limits to compute the limit.
6. **Mark up your graph** to indicate what the limit appears to be. Cut out this graph using scissors (or virtually), and paste or tape to the upper right hand corner of the printout from part 1. On this sheet, you should have a numerical, graphical, and algebraic determination of this limit.

Show work in a neat, precise manner. Put your name on all sheets and staple together. Make sure you clearly document each graph with the name of the function, the limit value (if it exists), and the reasons based on the graph of why this is the limit. You may scan and email these five problems to your instructor by the due date as a word document.

PLEASE DO ALL FIVE OF THESE PROBLEMS

	<u>Function $f(x)$</u>	<u>Enter into Limits.cgi</u>	<u>Point a</u>
1.	$2 + \sqrt{5x-8}$	<code>2 + sqrt(5x - 8)</code>	1.6
2.	$\frac{x-9}{\sqrt{x-3}}$	<code>(x - 9)/(sqrt(x) - 3)</code>	9
3.	$\begin{cases} x-2 & \text{if } x \leq 3 \\ 12-x^2 & \text{if } x > 3 \end{cases}$	<code>if (x<=3) {x-2} else {12-x^2}</code> (use the "dots" option on your graph).	3
4.	$\frac{x+6}{x^2+2x-24}$	use parentheses to separate numerator/denom.	-6
5.	$\frac{3\sin(x)}{x-k}$	Pick any integer value $k \neq 0$. Radians mode.	k (use same value)

Sample Problem from Lab #1

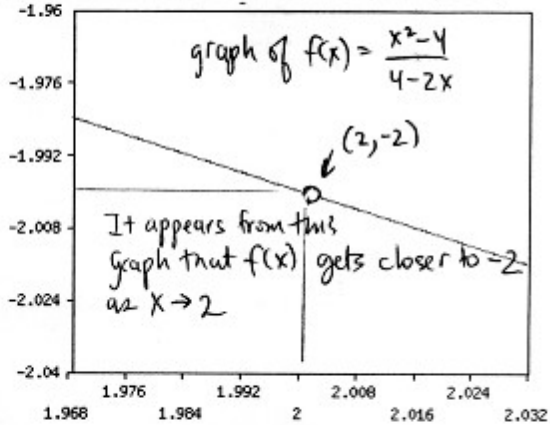
Function is $f(x) = (x^2 - 4)/(4 - 2x)$

Student: mead

Function is
$$f(x) = \frac{x^2 - 4}{4 - 2x}$$

Function evaluated near $x = 2$:

f(1) = -1.5
f(1.9) = -1.95
f(1.99) = -1.9949999999999999
f(1.999) = -1.9994999999999999
f(1.9999) = -1.9999500000000003
f(2) =
f(2.0001) = -2.0000499999999997
f(2.001) = -2.0005000000000007
f(2.01) = -2.0049999999999999
f(2.1) = -2.05
f(3) = -2.5



Evaluate $f(2)$.

$f(2)$ is undefined.

What conclusions can you draw about the limit as $x \rightarrow 2^-$ and as $x \rightarrow 2^+$. Why?

as $x \rightarrow 2^-$, it appears that $f(x) \rightarrow -2$

as $x \rightarrow 2^+$, it appears that $f(x) \rightarrow -2$.

What about the two sided limit as $x \rightarrow 2$? Why?

The two sided limit appears to exist because the left and right limits agree. $\lim_{x \rightarrow 2} f(x) = -2$.

Does the two sided limit as $x \rightarrow 2$ equal $f(2)$?

No. $f(2)$ doesn't exist.

Show algebraic justification of your conclusion on the limit, then attach graph

$$\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{4 - 2x} \right) \quad \text{plug in } x=2. \quad \frac{2^2 - 4}{4 - 2 \cdot 2} = \frac{0}{0} \text{ indeterminate.}$$

Try factoring.
$$\frac{(x+2)(x-2)}{-2(-2+x)} = \frac{x+2}{-2} \quad \text{plug in } x=2 \text{ again.}$$

$$\lim_{x \rightarrow 2} \left(\frac{x+2}{-2} \right) = \frac{2+2}{-2} = \frac{4}{-2} = \boxed{-2}$$

This is how I want each page to look like. Use scissors (virtual or real) to cut and paste the graph of $f(x)$ onto the limits.cgi printout.