

Calculus 1 - Test 3 - Review

Use this graph to fill in the blanks for a-g below:

1. a. $f(x)$ is increasing on the intervals _____.

b. $f'(x^3)$ is _____ (positive, negative, zero, or undefined).

c. $f''(x^6)$ is _____ (positive, negative, zero, or undefined).

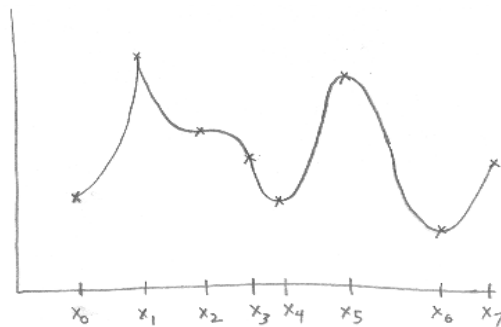
d. $f'(x)$ is undefined at _____.

e. The critical points for this function are _____.

f. Point(s) of inflection occur at _____ (use labeled points only).

f. The absolute minimum of this function occurs at _____.

g. This function has relative maxima at _____.



Answer the following questions:

h. (true or false) _____ Every critical point must also be a relative maximum or minimum.

i. $x = c$ is a critical point of $f(x)$ if _____ or _____.

j. The second derivative test for a critical point $x = c$ states that if:

if $f''(c) > 0$ then $x = c$ is a _____

if $f''(c) < 0$ then $x = c$ is a _____ and

if $f''(c) = 0$ or is undefined then _____

2. For $f(x) = x^3 - 3x^2 - 9x + 1$, find:

a) the intervals where $f(x)$ increases/decreases and relative max/min values.

b) $f(x)$ is concave up/down and point(s) of inflection.

c) graph $f(x)$, explicitly plotting and labeling any max/min and inflection points.

3. Given: $g(x) = \frac{2x(x-2)}{(x-3)^2}$ $g'(x) = \frac{-4(2x-3)}{(x-3)^3}$ and $g''(x) = \frac{4(4x-3)}{(x-3)^4}$

- a. compute the x-intercept(s)
 - b. compute the vertical asymptote(s)
 - c. compute the horizontal asymptote(s) and show all work!
 - d. find all critical points
 - e. use the second derivative test to determine if each critical point is a max or min. (disregard critical values for which f(x) is undefined)
4. Sketch the graph of f(x) if f(x) is a rational function and
- the zeros of f(x) are -2, 1 and 4
 - the vertical asymptote is at x = 2 and the horizontal asymptote is at y = 1.
 - critical values are x = -1/2 and x = 2, f(-1/2) = -2 and f'(-1/2) = 0.
 - f(x) decreases on (-∞, -1/2) and increases on (-1/2, 2) and (2, ∞).

5. a. Find $\lim_{x \rightarrow -\infty} \frac{2x^2 + 5}{5x^3 - 10x}$ Show all work.

b. Compute $\lim_{x \rightarrow \infty} \frac{(4x+5)(2x+3)}{10-5x^2}$

6. Given $x^3 - y^2 = 3y - 10$. If $\frac{dy}{dt} = 4$ when $y = 3$, find $\frac{dx}{dt}$

7. An upward facing right circular cone is being filled with water at a rate of 48π cubic feet per minute. If the radius r of the water in the cone is always equal to its height h , then how quickly is the radius r of the water changing when $r = 4$. Note: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$