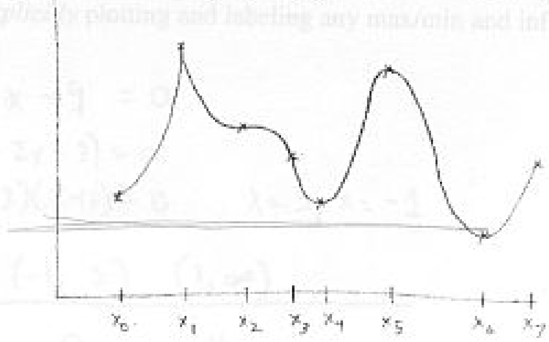
Calculus 1 – Test 3 –Spring 2001 Name Answers

[20] *1. Use the information on this graph to fill in the blanks (a – g) below.*



a. f(x) is increasing on the intervals (x_0, x_1) (x_4, x_5) (x_6, x_7).

b. f '(x_3) is negative (positive, negative, zero, or undefined).

c. f ''(x_6) is positive (positive, negative, zero, or undefined).

d. f '(x) is undefined at x_1.

e. The critical points for this function are x_1, x_2, x_4, x_5, x_6.

f. Point(s) of inflection occur at x_2, x_3, (use labeled points only).

f. The absolute minimum of this function occurs at x_6.

g. This function has relative maxima at x_1, x_5.

Answer the following questions:

h. (true or false) false Every critical point must also be a relative maximum or minimum.

i. x = c is a critical point of f(x) if $f'(c) = 0$ or undefined.

j. The second derivative test for a critical point x = c states that if:

if f ''(c) > 0 then x = c is a min

if f ''(c) < 0 then x = c is a max and

if f ''(c) = 0 or is undefined then we don't know

[20] 2. For $f(x) = x^3 - 3x^2 - 9x + 1$, find:

a) the intervals where f(x) increases/decreases and relative max/min values.

b) f(x) is concave up/down and point(s) of inflection.

c) graph f(x), *explicitly* plotting and labeling any max/min and inflection points.

$$f'(x) = 3x^2 - 6x - 9 = 0$$

$$3(x^2 - 2x - 3) = 0$$

$$3(x-3)(x+1) = 0 \qquad x = 3,\ x = -1$$

	$(-\infty,-1)$	$(-1,3)$	$(3,\infty)$
x	-2	0	4
$f'(x)$	$+$	$-$	$+$

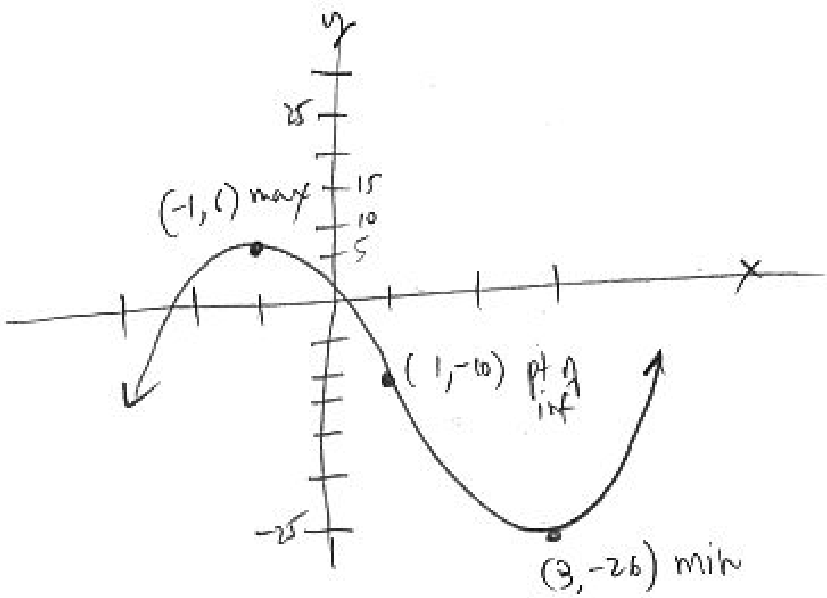
$f(-1)$ is a max $\quad f(-1) = 6$

$f(3)$ is a min $\quad f(3) = 27 - 27 - 27 + 1 = -26$

$$f''(x) = 6x - 6 = 0 \qquad x = 1$$

	$(-\infty,1)$	$(1,\infty)$
x	-2	0
$f''(x)$	$-$	$+$

$f(1) = 1 - 3 - 9 + 1 = -10$ is a pt. of inf.



[15] 3. Given: $g(x)=\frac{2x(x-2)}{(x-3)^2}$ $g'(x)=\frac{-4(2x-3)}{(x-3)^3}$ and $g''(x)=\frac{4(4x-3)}{(x-3)^4}$

a. compute the x–intercept(s)

$\{0, 2\}$ because $2x(x-2)=0$ where

$2x=0 \mid x-2=0$

$x=0 \quad 2$

b. compute the vertical asymptote(s)

$\{3\}$ because $(x-3)^2=0$ where $x-3=0$

c. compute the horizontal asymptote(s) and show all work!

foil out take lim at $\pm\infty$.

$$\frac{2x^2-4x}{x^2-6x+9} = \frac{2-4/x}{1-6/x+9/x} \qquad \lim_{x\to\pm\infty} \frac{2-4/x}{1-\frac{6}{x}+\frac{9}{x}} = \frac{2-0}{1-0+0}$$

$$\frac{2}{1}=2.$$

d. find all critical points (first deriv.)

top: $2x-3=0$ when $x=3/2$

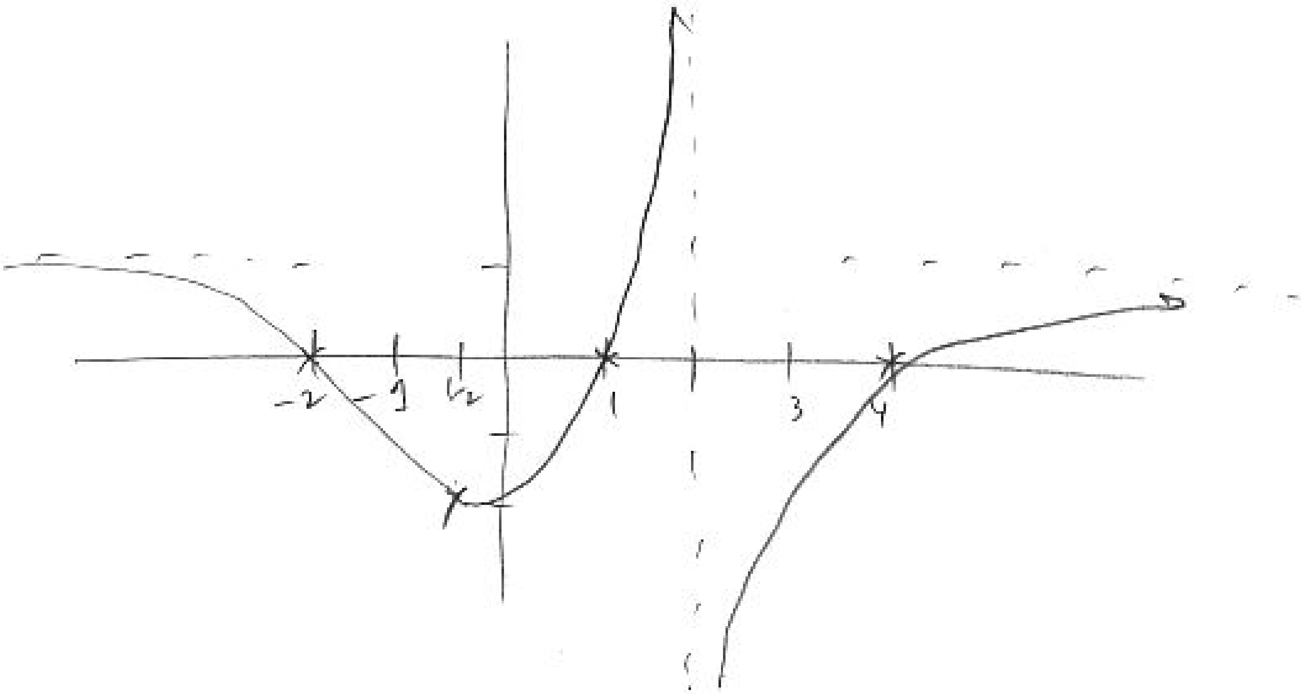
bottom: $(x-3)^2=0$ when $x=3$

e. use the second derivative test to determine if each critical point is a max or min. (disregard critical values for which f(x) is undefined)

$$g''\left(\tfrac{3}{2}\right) = \frac{4\left(4\cdot\frac{3}{2}-3\right)}{\left(\frac{3}{2}-3\right)^4} = \frac{4(3)}{+} = + \quad \text{min}.$$

[10] 4. Sketch the graph of f(x) if f(x) is a rational function and

- the zeros of f(x) are −2, 1 and 4
- the vertical asympote is at x = 2 and the horizontal asymptote is at y = 1.
- critical values are x= −1/2 and x = 2, and f(−1/2) = −2.
- f(x) decreases on (−∞, −1/2) and increases on (−1/2, 2) and (2, ∞).



[10] 5. a. Find $\lim\limits_{x \to -\infty} \dfrac{2x^2+5}{5x^3-10x}$ Show all work.

$$\lim_{x \to -\infty} \frac{2x^2+5}{5x^3-10x} = \frac{2/x + 5/x^3}{5 - 10/x^2} = \frac{0+0}{5-0} = \underline{0}$$

5(b) $\lim\limits_{x\to\infty} \frac{(4x+5)(2x+3)}{10-5x^2} = \lim\limits_{x\to\infty} \frac{8x^2+22x+15}{10-5x^2}$ divide by x^2

$= \lim\limits_{x\to\infty} \frac{8+\frac{22}{x}+\frac{15}{x^2}}{\frac{10}{x^2}-5}$ take limit of each term

$= \frac{8+0+0}{0-5} = -\frac{8}{5}$

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6. $x^3 - y^2 = 3y - 10$ when $y=3$, $x^3-(3)^2 = 3(3)-10$

$x^3 - 9 = 9 - 10$

and $x^3 = 8$ $x=2$

$\frac{d}{dt}x^3 - \frac{d}{dt}y^2 = \frac{d}{dt}3y - \frac{d}{dt}10$

$3x^2\frac{dx}{dt} - 2y\frac{dy}{dt} = 3\frac{dy}{dt} - 0$ when $x=2$, $y=3$, we know $\frac{dy}{dt}=4$

$3(2)^2\left(\frac{dx}{dt}\right) - 2(3)(4) = 3(4) \Rightarrow 12\frac{dx}{dt} - 24 = 12$ So $\frac{dx}{dt} = 3$

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7. $V = \frac{1}{3}\pi r^2 h$ and we know that $h=r$ for the whole problem,

So $V = \frac{1}{3}\pi r^2 r = \frac{1}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \cancel{\frac{1}{3}}\pi\left(\cancel{3}r^2\frac{dr}{dt}\right)$

so $\frac{dV}{dt} = \pi r^2 \frac{dr}{dt}$ ...

we're given that $\frac{dV}{dt} = 48\pi$, so $48\cancel{\pi} = \cancel{\pi}(4)^2\frac{dr}{dt}$
when $r=4$

$\frac{dr}{dt} = 3$ ft/min